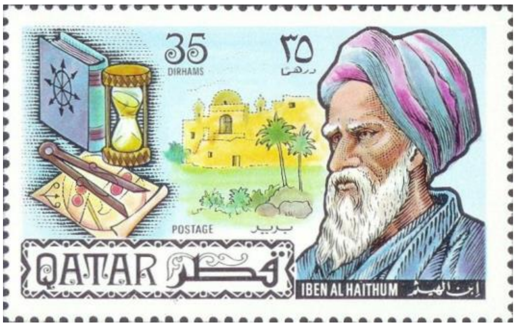
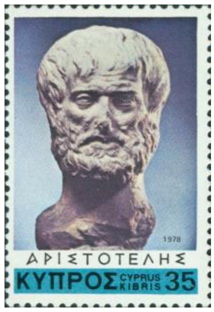
IBN AL-HAITHAM (965 – 1039)

by HEINZ KLAUS STRICK, Germany



ABU ALI AL-HASAN IBN AL-HAITHAM is also known in Europe as ALHAZEN (= AL-HASAN). He is considered one of the most important universal scholars of the Islamic Middle Ages. AL-HAITHAM wrote over 200 works in which he dealt with mathematical, medical, philosophical and especially physical questions.

There is varying and contradictory information about the individual phases of his life. In one of his writings, which contains autobiographical references, he "only" addresses his irritations regarding various religious doctrines. In his birthplace of Basra (then Persia, now Iraq), he began a civil service career that led him to the office of *vizier* (minister). However, this job did not satisfy him.



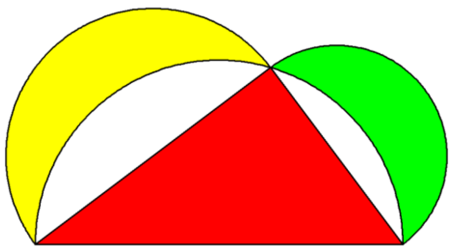
He succeeded in feigning a certain "insanity" which freed him from administrative activities and gave him the opportunity to occupy himself intensively with the writings of ARISTOTLE and to begin his own scientific studies. He dealt with the conic sections, the problem of squaring the circle and the trisection of an angle, as well as the construction of a regular heptagon.

Meanwhile, the *Fatimids* (a religious Islamic movement based on FATIMA, the daughter of the Prophet MOHAMMED) established a new empire in North Africa and founded Cairo as its capital.

The second caliph of the ruling dynasty, AL-HAKIM, was told of AL-HAITHAM's abilities and he invited him to come to Egypt to regulate the Nile floods. AL-HAITHAM accepted the caliph's invitation, but after an expedition to the cataracts in Aswan, he realised that even he could not succeed in doing what the ancient Egyptians had not been able to do: i.e. that regulating the Nile was not possible.

AL-HAKIM was very disappointed by this negative feedback from AL-HAITHAM, but nevertheless offered him an administrative post. AL-HAITHAM quickly realised how dangerous and unpredictable the ruler was. Once again he resorted to the trick of "mental derangement". He was placed under house arrest and although he was deprived of access to his assets, the trick guaranteed him physical safety and the almost unrestricted possibility to conduct research.

After AL-HAKIM's death in 1021, he was finally able to return to public life – after his miraculous "recovery", he got his fortune back and travelled to Syria and Spain (at that time under Arab rule). He then earned his living as a teacher at the *University of Cairo*, but also as an expert translator of ancient mathematical, physical and medical texts.



In geometry, AL-HAITHAM tried in vain to determine the area of a circle (i.e. the number π) by calculating the intersections of circles ("lunes").

He also considered a quadrilateral with three right angles and proved that the fourth angle can be neither an acute nor an obtuse angle. This was his attempt to "prove" the parallel axiom of EUCLID.

In number theory, he formulated a problem: he was looking for a number that leaves the remainder 1 after division by 2, 3, 4, 5 and 6, but which was supposed to be divisible by 7.
He realised that there were an infinite number of solutions to this problem and that 6! + 1 was a possible solution.

The problem can be generalised. The statement

- *If p is a prime number, then* $1+(p-1)!$ *is divisible by p.*

is today often called WILSON's theorem (after the English mathematician JOHN WILSON, 1741-1793).

EUCLID had proved:

- *If* $2k-1$ *is a prime number, then* $2^{k-1}\cdot(2^k-1)$ *is a perfect number.*

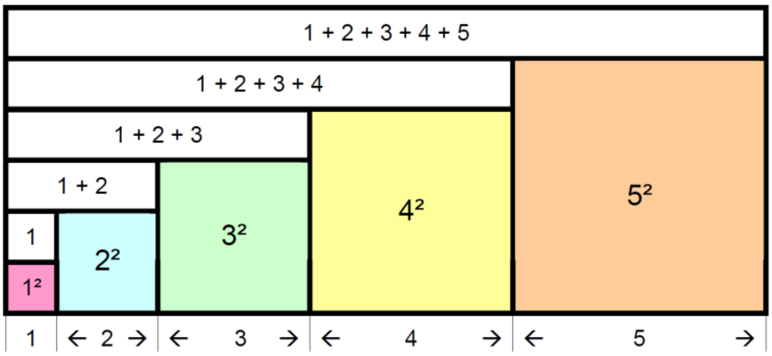
AL-HAITHAM stated that the converse of this theorem also holds. Much later, LEONHARD EULER succeeded in proving this for *even* perfect numbers.

That the sum of successive cubic numbers is equal to the square of the sum of the numbers themselves was already known to Indian mathematicians in the 5th century:

$$1^3+2^3+\ldots+n^3=(1+2+\ldots+n)^2=\left(\tfrac{n(n+1)}{2}\right)^2$$

AL-HAITHAM found a way to derive formulae for higher power sums. He started with a rectangular figure showing that

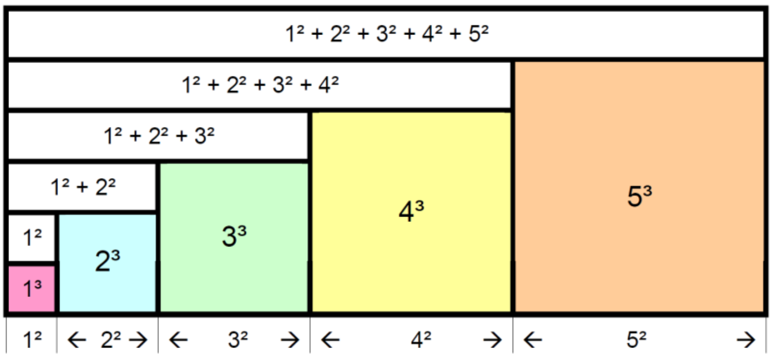
$$\left(1^2+2^2+3^2+\ldots+n^2\right)+(1)+(1+2)+(1+2+3)+\ldots+(1+2+3+\ldots+n)=(1+2+3+\ldots+n)\cdot(n+1)$$



and thus derived a formula for the sum of the first *n* square numbers

$$1^2+2^2+3^2+\ldots+n^2=\tfrac{1}{3}n^3+\tfrac{1}{2}n^2+\tfrac{1}{6}n\,.$$

With the same figure but interpreted differently, namely that the squares should represent rectangles with heights 1, 2, 3, … and "widths" 1^2, 2^2, 3^2, …, that is, the "squares" have the area 1^3, 2^3, 3^3, …, he established a connection between the sum of 3rd powers and various sums of lower powers.



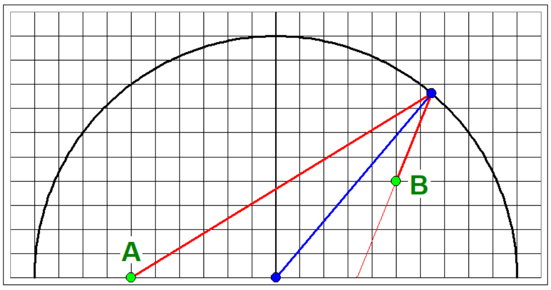
With the same trick, he could also derive a formula for the sum of 4^{th}, 5^{th} and 6^{th} powers.

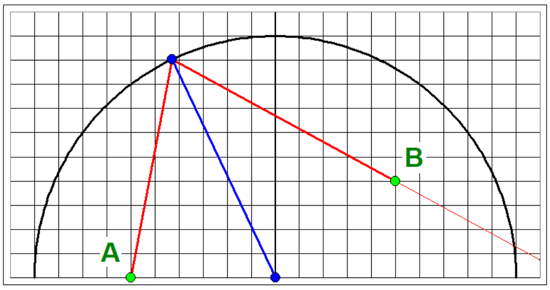
His most important scientific contribution was a 7-volume work on optics (*Kitab al-Manazir* – Treasure of Optics), which was translated into Latin in 1270 as *Opticae thesaurus Alhazeni* and had a decisive influence on the development of physics in the West.

Particularly noteworthy was his "modern" scientific approach, based on repeatable experiments rather than speculative theories or subjective opinions; when he made a hypothesis, he in turn verified it by experiment.

In *Kitab al-Manazir* one finds, among other things, a problem that has entered the literature as ALHAZEN's problem (also known as ALHAZEN's billiard problem):

- *In a circular area two points A and B are given. Which points of the circular line can be connected to the two points so that the bisector of the resulting angle passes through the centre of the circle?*





Described as a problem in optics:

- *At which point of a circular mirror must an observer B standing inside the circle look in order to see the reflected image of an object A that is also inside the circle?*

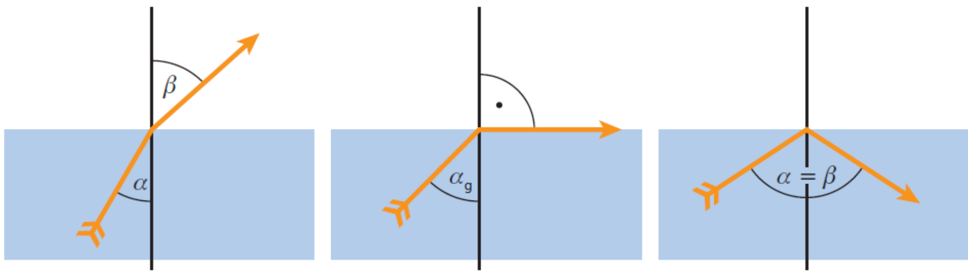
As a billiard problem:

- *How do you have to play a ball A on a circular billiard table against the cushion so that a ball B is hit?*

The points sought on the circular line (in general, four solutions exist) cannot be constructed with compass and ruler; the algebraic solution leads to a 4th degree equation.

In this work, AL-HAITHAM also described the physical structure of the eye, explained how the *camera obscura* (pinhole camera) works and stated that vision functions by objects reflecting light (and not by rays emanating from the eye and thus capturing the observed objects, as PTOLEMY and EUCLID assumed). He established the laws of reflection and described the construction of reflected rays in plane, spherical, cylindrical and parabolic surfaces.

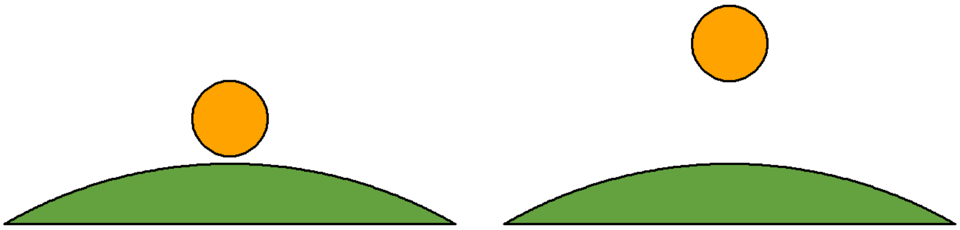
In his description of the phenomenon of light refraction when passing from one medium to another, he made use of the idea that the (light) velocity is smaller in the denser medium. He experimentally determined the critical angle of *total reflection*.



AL-HAITHAM described how curved glasses (lenses) work. After the translation of his main work into Latin, monks developed *reading stones* (spherical glass bodies), the forerunners of magnifying glasses and spectacles.

He explained why there is twilight in the morning and in the evening (refraction of light at the earth's atmosphere), and estimated a *height* of the earth's atmosphere of about 15 km from the limiting angle of 19° determined in the process.

He dealt with the phenomenon that the sun and the moon appear larger near the horizon and proved by measurement that this is only an illusion.



He conducted experiments to investigate the dispersion of sunlight. All this impressively justifies why he is called the *father of optics*.

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https://www.spektrum.de/wissen/abu-ali-al-hasan-ibn-al-haitham-965-1039/893657

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*) The present text has been extended by some details taken from *Mathematics is beautiful* (Springer 2021)