MOHAMMED AL KHWARIZMI (780 – 850)

by HEINZ KLAUS STRICK, Germany

From 786 to 809, Caliph HARUN-AL-RASHID reigned from Baghdad over a world empire stretching from Spain to the foothills of the Himalayas. Under his reign, the arts and sciences were promoted and a diverse cultural life developed at his court.

His successor on the throne – after a bloody power struggle – was his younger son AL-MAMUN. He founded the *House of Wisdom* in Baghdad as the centre of science. A comprehensive library was established there – comparable to the library in Alexandria; important works from other cultures were translated into Arabic, including those of Greek philosophers and mathematicians.

Among the important scholars of the House of Wisdom were ABU JA'FAR MOHAMMED IBN MUSA AL-KHWARIZMI, ABU YUSUF YAQUB IBN ISHAQ AL-SABBAH AL-KINDI and the three Bana Musa brothers: JAFAR MUHAMMAD, AHMED and AL-HASAN IBN MUSA IBN SHAKIR.

MOHAMMED AL-KHWARIZMI's place of birth or exact dates are not known though possibly his family originated from the Persian city of Khwarizm (today in Uzbekistan).

We owe to him the adoption of the Indian numerals, which since then have been called *Arabic-Indian numerals*. He recognised the advantages of decimal notation – especially the role of *zero* as a placeholder for unoccupied digits in the place value system.

AL-KHWARIZMI referred to the zero as *as-sifr* (the void), from which the words like 'zero', 'zefiro', 'cipher', 'Ziffer' are derived in various European languages.

His main work is entitled *Al Kitab al-muhtasar fi hisab al-gabr w-al-muqabala* (A concise book on the calculation methods by addition and compensation).

- *al-gabr* (= supplement) means: On both sides of an equation, equal terms are added up to produce an equation in which a sum occurs from an equation in which there is a difference: $ax^2 - bx = c$ is transformed into $ax^2 = c + bx$.

In the 12th century the work was translated into Latin (*Liber algebrae et almucabala*).

The term *algebra* was thus created in Europe from the translation of the book title.

The name of the mathematician himself is also the origin of a word in our modern language: the Latin translation of his book on Indian numerals is entitled *Algoritmi de numero Indorum*; and it was quoted as *Algorizmi dixit*.

So from the name of AL-KHWARIZMI we get the term "algorithm" for a calculation method.

What distinguishes this book is its systematic structure – not only are problems strung together and solved, but they are reflected on:

First of all, all possible types of linear and quadratic equations were listed; everything had to be expressed in words – a formal notation of the kind we take for granted today had not yet been developed. A distinction was made between the square (x^2), the root (x) and numbers. This was how the different cases arise:

(1) $ax^2 = bx$ (*squares equal to roots*)

(2) $ax^2 = b$ (*squares equal to numbers*)

(3) $ax = b$ (*roots equal to numbers*)

and analogously to describe

(4) $ax^2 + bx = c$ (5) $ax^2 + c = bx$ (6) $ax^2 = bx + c$

The coefficients a, b, c quoted here represent positive numbers. If differences occur in an equation, the equation must first be brought to one of the above-mentioned forms by *al-gabr*.

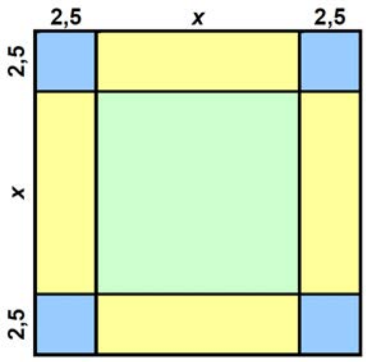
AL-KHWARIZMI interpreted the equations as geometrical problems; therefore only positive solutions existed for him. It is therefore not surprising that he gave the same solutions for equations (1) and (3) (zero was not accepted as a solution to an equation until the 17th century).

AL-KHWARIZMI explained the methods of solution by means of concrete numerical examples.

- Solution of the equation $x^2 + 10x = 39$ (*squares and roots equal to numbers*)

First of all we consider a square of side length x (highlighted in green).

We add four rectangles with the side lengths 5 and x (yellow); this figure has a total area of $x^2 + 10x$, i.e. 39.



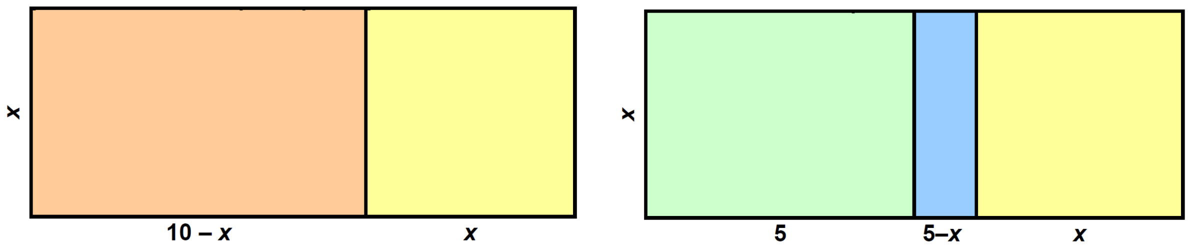
If we add a square to the figure (*square addition*), this is made up of the inner square, the yellow coloured rectangles and four small blue squares of side length 2.5 (i.e. each with area 6.25).

The area of the new square figure is therefore: $39 + 4 \cdot 2.5^2 = 64$.

The side length of the large square is therefore 8, which means that the inner square must have a side length of $x = 3$.

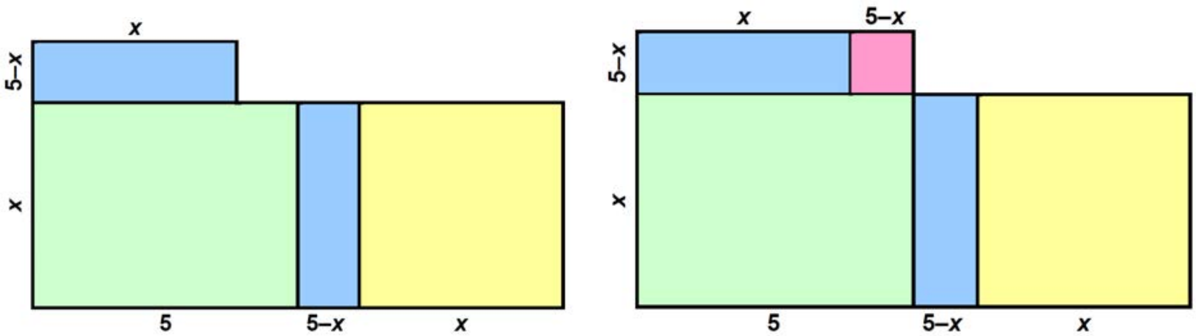
- Solution of the equation $x^2 + 21 = 10x$ (squares and numbers equal to roots):

We look at a rectangle with sides 10 and x, i.e. with a surface area $10x$; on the right, we cut off a square (highlighted in yellow) with a surface area x^2 ; the remaining rectangle (orange) has a surface area of 21.



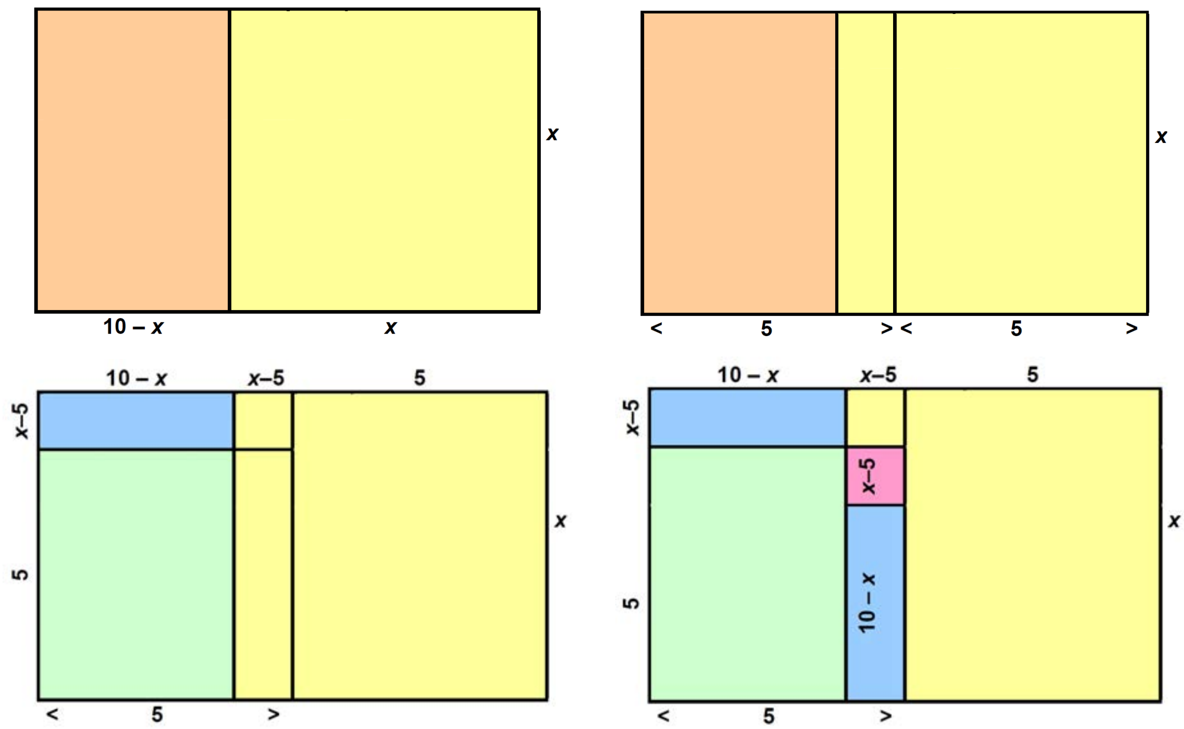
We then halve the base of the original rectangle and complete the left half of the rectangle (green) to form a square (side length 5, i.e. with a surface area 25).

This square then consists of three parts: a rectangle (green) with the side lengths 5 and x, a rectangle (blue) with the side lengths x and $5-x$ at the top, and a small square (pink) with the side length $5-x$, i.e. the area $(5-x)^2$.



Since the green-coloured and the blue-coloured area together have an area of 21 and the large square has an area of 25, the relationship $(5-x)^2 = 4$ holds for the square at the top right. This means that the unknown length x must equal 3.

If one chooses the side length x greater than 5 in the drawing, a figure is obtained which leads to the equation $(x-5)^2 = 4$ and thus to the second solution $x = 7$.



Furthermore, one can see that equations such as $x^2 + 26 = 10x$ cannot have a solution.

Hisab al-gabr w-al-muqabala contained not only this systematic theory of equations but also a reflection on what numbers are and what rules can be applied to calculating with terms, e.g. how to multiply terms of type $a + bx$ by each other and how to determine measures of areas and volumes (circle, sphere, cone, pyramid).

A large part of Al-Khwarizmi's book is devoted to tasks designed to solve difficult everyday problems, including those arising from the application of *Islamic law*.

Example: A woman dies and leaves her inheritance to her husband, son and three daughters; a seventh and an eighth of the assets belong to another person. Under *Islamic law*, the husband inherits a quarter of the remainder, and the male descendants inherit twice as much as the daughters.

Al-Khwarizmi also wrote several treatises on astronomy with tables of the movement of the sun, moon and planets – the basis of many subsequent works by astronomers of the Islamic-Arabic cultural sphere and Europe. He invented devices for observing the sky and further developed sundials so that devout Muslims could more easily read the time for the prescribed prayer. Other writings dealt with the problem of designing calendars that take proper account of the movement of the sun and moon.

Based on the geography of Claudius Ptolemy (100 – 175), Al-Khwarizmi, in his book *Kitab Surat al-ard* (Picture of the Earth), compiled the latitude and longitude coordinates of 2402 cities, mountains, islands, etc. in the world known at that time. He corrected Ptolemy's data concerning the east-west extension of the Mediterranean Sea (50 instead of 63 degrees of longitude) and the longitude through the "shore of the Western Ocean" became the zero meridian of his *Earth coordinate system*.

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