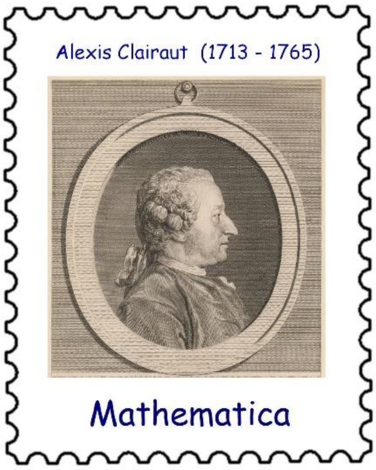
Alexis-Claude Clairaut (May 7, 1713 – May 17, 1765)

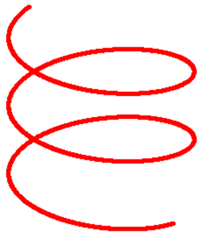
by Heinz Klaus Strick, Germany



That mathematics would play an important role in his life was evident in him from an early age: Alexis-Claude Clairaut, second child of Jean-Baptiste Clairaut and Catherine Petit, is reported to have learned to read by studying Euclid's *Elements*. (His father was a mathematics teacher at a school in Paris and a corresponding member of the *Berlin Academy of Sciences*).

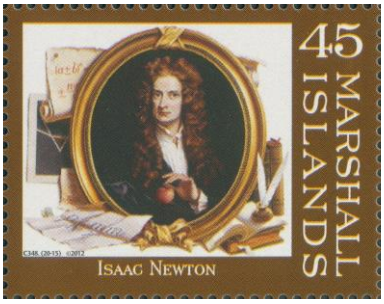
When Alexis was nine years old, his father gave him a demanding book on geometry and algebra to read, which also contained an introduction to differential and integral calculus as well as analytical geometry. The following year, his reading list includes Guillaume de L'Hôpital's *Analyse des infiniment petits pour l'intelligence des lignes courbes*, the book on Leibniz's differential calculus published in 1696.

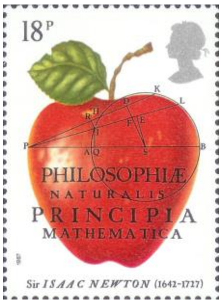
At the age of 13, the boy wonder was invited to present his investigations *Quatre problèmes sur de nouvelles courbes* at the *Académie Royale des Sciences* – about four curves he had discovered. (A few years later, one of his younger brothers was also allowed to present there – however, he is then "already" 14 years old ... This brother died at the age of 16, and Alexis was finally the only one of a total of 20 children of the family who survived childhood).

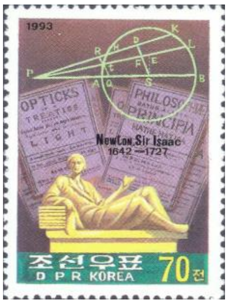


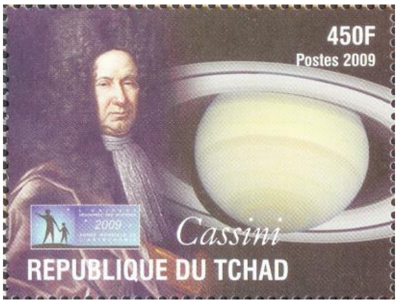
In 1729, Alexis Clairaut wrote a paper of over 100 pages, *Recherches sur les courbes a double courbure*, in which he investigated curvature and torsion (= *double courbure*) of curves in space. The members of the *Académie* then proposed him for admission to the *Association of Scientists*, but the King, however, only agreed when Clairaut had turned 18 – before him and after him, no other member had been appointed at such an early age.

Clairaut befriended Pierre Louis Moreau de Maupertuis, 15 years his senior, who regularly gathered a group of younger scientists around him. These pursued the goal of also convincing the French scientists of the correctness of Newton's doctrine regarding the existence of a gravitational force – as presented in the *Principia*.





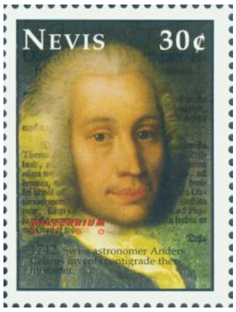




Clairaut became increasingly fascinated by the question of what shape the Earth had.

At the *Académie*, he followed the dispute between Newton's followers, who were convinced that the Earth was flattened at the poles, and the group around Jacques Cassini (II), who supported René Descartes' theory, namely that the Earth had a shape tapering towards the poles.

Since the surveying work of JEAN-DOMINIQUE CASSINI and his son JACQUES had not led to a clarification of this question within France, the *Académie* decided to send two expeditions, one to Peru and one to northern Europe, to determine the distance between two circles of latitude with the help of triangulations: If the Earth is flattened towards the poles, the distance between two circles of latitude would be smaller at the equator than near the poles, and vice versa.



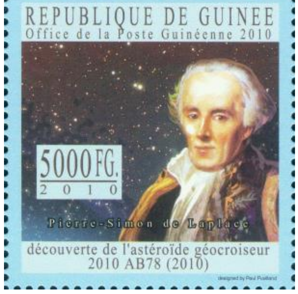
In 1736, MAUPERTUIS was commissioned to lead the expedition to Lapland to determine the distance between the 66th and 67th parallel and in addition to CLAIRAUT, the Swedish explorer ANDERS CELSIUS was also a member of the expedition.

The measurements near the pole were completed within 16 months but the participants of the South America expedition led by CHARLES MARIE DE LA CONDAMINE, on the other hand, returned to France only after ten years.

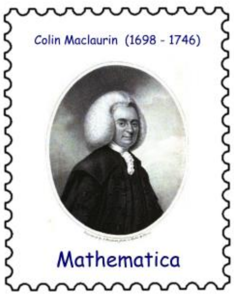
On his way back, CLAIRAUT wrote a paper for the *Royal Society* in London in which he confirmed that the shape of the Earth corresponded to NEWTON's assumption, but that NEWTON's reasons for his assumption were wrong.

In 1743, he then published the treatise *Théorie de la figure de la terre*, in which he gave a formula with which one could calculate the gravitational acceleration $g(\varphi)$ as a function of the respective latitude φ: $g(\varphi) \approx g_{eq} \cdot [1 + (\frac{5}{2}m - f) \cdot \sin^2(\varphi)]$ where $f = \frac{a-b}{a}$ is the *flattening coefficient* with respect to the length of the semi-axes *a* and *b* of the rotational ellipsoid, g_{eq} is the acceleration measured at the equator and *m* is the ratio of centrifugal and gravitational forces at the equator.

Forty years later, PIERRE-SIMON LAPLACE determined the value $f \approx \frac{1}{330}$ with the help of CLAIRAUT's formula (based on measurements at 15 points on the Earth) – today, the value $f \approx \frac{1}{298{,}2}$ is assumed to be correct.



CLAIRAUT's calculations were based on a model of the Earth for which the Scottish mathematician COLIN MACLAURIN received a prize from the *Académie* in 1740 to explain the phenomenon of ebb and flow of the tides: According to this model, the Earth's sphere is composed of homogeneous shells concentric with each other, which take on the shape of an ellipsoid due to the Earth's rotation.



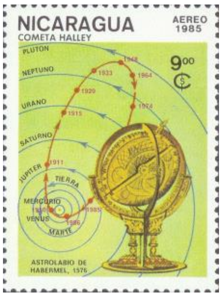
After completing his investigations into the shape of the Earth, CLAIRAUT threw himself into solving the next problem. The orbit of the Earth around the Sun and that of the Moon around the Earth can only be described approximately with the help of KEPLER's laws and NEWTON's law of gravitation.

In his investigations, CLAIRAUT even came to the conclusion that NEWTON's law was wrong and had to be supplemented by an additive correction element in the equation, which he announced at the *Académie* at the end of 1747. After extensive calculations lasting months, however, he found that the observed data were in accordance with the theory and that the deviations that had occurred previously were only due to a lack of computational accuracy.

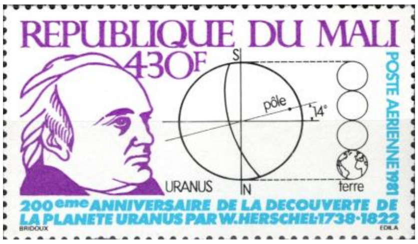
LEONHARD EULER in faraway St Petersburg, who had also tried to resolve the apparent contradictions, wanted to learn more about CLAIRAUT's calculations. Therefore, in 1752, he had the St Petersburg Academy announce a competition to find a method to calculate the farthest point of the moon's orbit (*apogee*). With unmistakable envy, he judged CLAIRAUT's competition entry *Théorie de la lune*: "The most important and profound discovery ever made in mathematics."

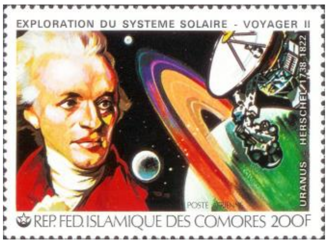
Encouraged by this success, CLAIRAUT turned his attention to the return of HALLEY's comet. In November 1758, with even greater mathematical effort, he determined 15 April 1759 as the time of the closest point to the sun (*perihelion*) of the comet's orbit. HALLEY himself had predicted the month of December 1758 in 1705. The fact that it was actually 13 March no longer mattered: CLAIRAUT was celebrated in the *Académie* as the "second THALES".

In another competition entry for the St Petersburg Academy in 1762, he improved his calculation methods by also taking into account the influence of the planets Jupiter and Saturn on the comet's orbit (*Recherches sur la comète*).



The fact that he did not arrive at 13 March in spite of these additional considerations was due to the existence of another planet. Uranus, the seventh planet in our solar system, was actually discovered by WILLIAM HERSCHEL in 1781.





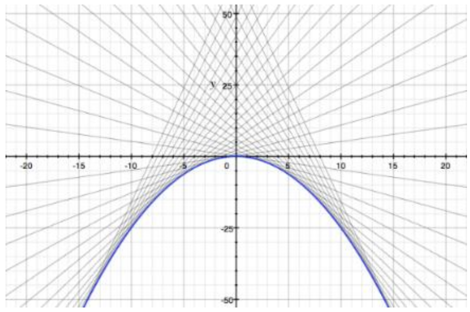
CLAIRAUT was now at the peak of his career and in the meantime, he had been accepted as an honorary member by the Academies in London, Berlin, St Petersburg, Bologna and Uppsala.

Due to his scientific successes, he could hardly save himself from invitations in his private life and he neglected his health. None of his numerous affairs led to a lasting relationship and thus he remained unmarried. In 1765, after a short illness, he died at the age of (only) 52.

CLAIRAUT's life's work includes a number of other contributions:

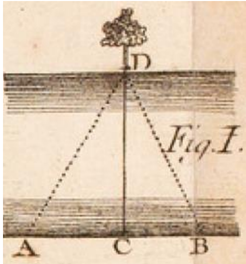
- In the 1730s he published various works on differential and integral calculus: In *Sur quelques questions de maximis et minimis* (1733) he developed methods in the so-called calculus of variations.

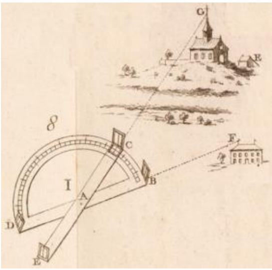
- In 1734, one finds in the minutes of the *Académie* his solution for (today, so-called) *CLAIRAUT differential equations* of the type $y(x) = x \cdot y'(x) + f(y'(x))$.
  These can be solved by differentiation: From $y'(x) = y'(x) + x \cdot y''(x) + f'(y'(x)) \cdot y''(x)$, i.e. $[x + f'(y'(x))] \cdot y''(x) = 0$, follows $y''(x) = 0$ or $x + f'(y'(x)) = 0$.
  The former leads to $y'(x) = c$. If you insert this into the initial equation, you get the so-called *general solution* $y(x) = x \cdot c + f(c)$, i.e. a set of linear functions. On the other hand, $x + f'(y'(x)) = 0$ results in the so-called *singular solution*, whose graph is the envelope of the linear functions.

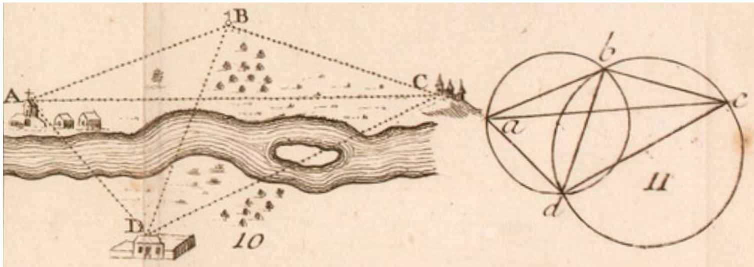


- In a paper from 1739, one finds a reference to a discovery that is usually referred to in the literature as the *theorem of SCHWARZ* (HERMANN AMANDUS SCHWARZ, 1843-1921): In the case of multiply (continuously) differentiable functions of several variables, it does not matter in which order the individual variables are differentiated.

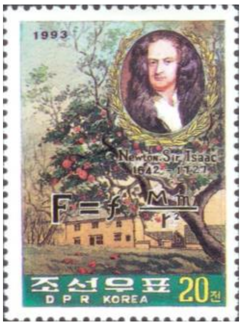
- Since CLAIRAUT was dissatisfied with the geometry books that were widespread in his time, which – in the style of EUCLID's *Elements* – were essentially concerned with proofs of theorems and constructions and less with the application of geometry, he published the textbook *Élémens de Géometrie* in 1741. Already the introduction to the first chapter made clear what should be emphasised more in school: With the help of elementary methods, lengths of distances and areas could be determined. The book of over 200 pages contained numerous illustrations, including the ones below, which can hardly be found in other geometry books of the time. In the *Élémens d'Algèbre*, published in 1749, CLAIRAUT also endeavours to convince the readers of his book of over 350 pages of the usefulness of algebra by means of extensive commentaries.







- Finally, Clairaut's collaboration in the translation of the *Principia* by Émilie du Châtelet should not go unmentioned. In 1756, seven years after her death, the text, supplemented by Clairaut's commentaries, was published and the importance of Newton's work was recognised on the continent.



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