GEORGE GREEN (July 14, 1793 – May 31, 1841)

© Green's Windmill & Science Centre, Nottingham

Since the 18th century, special honours that can be bestowed posthumously on a citizen of Great Britain have included the erection of a monument or a plaque in *Westminster Abbey*. Here, near the imposing tomb of ISAAC NEWTON, you can find tablets dedicated to WILLIAM THOMSON (LORD KELVIN), MICHAEL FARADAY, JAMES CLERK MAXWELL, WILLIAM and JOHN HERSCHEL, GEORGE GABRIEL STOKES and GEORGE GREEN. The commemorative plaque for GREEN shown on the Mathematica stamp on the right was only installed in 1993 – on the occasion of his 200th birthday. There were special reasons why this happened so late, which had to do with the living conditions of the generally little-known genius, but also with the fact that he had already died by the time his contributions were understood.

GEORGE GREEN's exact date of birth is unknown and only the date of his baptism, 14 July 1793, was recorded. His father, whose first name was also GEORGE, and his mother SARAH ran a successful bakery in Sneinton (now a suburb of Nottingham). In 1800, when food prices rose dramatically as a result of the blockade measures by Napoleonic France, there were serious losses due to looting and burglaries.

Attending school was not yet common in the lower classes of society at that time. As GEORGE at the age of eight showed a particular interest in arithmetic, his father saw fit to enrol him in ROBERT GOODACRE'S prestigious (and expensive) private school. GOODACRE knew how to make his lessons interesting and stimulate children's curiosity by demonstrating experiments. Later, travelling around Great Britain and the USA, he earned his living by giving popular scientific lectures.

GEORGE's school attendance was limited to just one year, which was quite common. It is said that his teacher could not have taught him anything more in mathematics. In any case, the short time at school was enough for the boy to develop a special interest in mathematics and physics.

However, there was initially little time in GEORGE'S daily routine to pursue this interest. Although he was only nine years old after his short school attendance, he already had to work regularly in his father's bakery.

Thanks to his flourishing business, the father was able to acquire a large fortune in the following years, with which he bought several houses and a property outside Sneinton. On this site he had a brick windmill built – a special feature at this time – which is now one of Nottingham's sights – GREEN's Mill. In 1817, GREEN (senior) built a house next to it, in which his 24-year-old son also lived, while his sister ANN, who was two years younger, married a cousin and moved away.

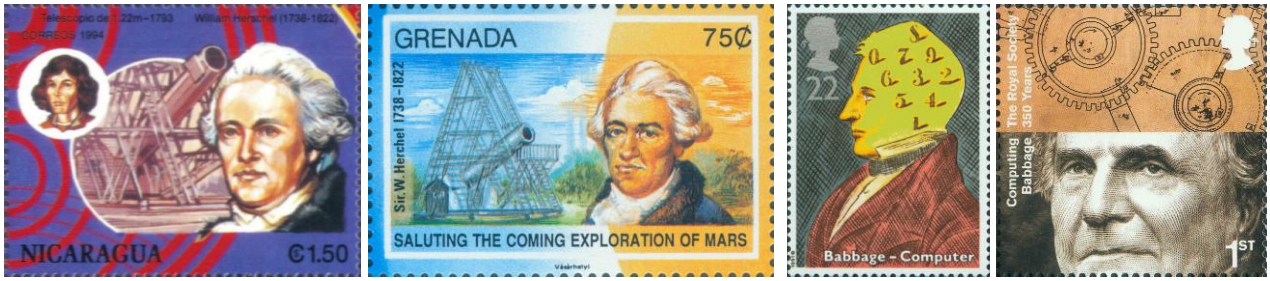
There is no concrete evidence, but GEORGE GREEN (junior) must have been intensively involved in mathematics in addition to his work as a baker and miller. There is much to suggest that he received inspiration and appropriate reading through close contact with JOHN TOPLIS, a scholar who lived in the neighbourhood. TOPLIS had completed his studies at *Queen's College* in Cambridge in 1801 with brilliant grades and then – dissatisfied with the mathematics that continued to be taught in the NEWTONIAN tradition – occupied himself with the latest developments in France. For example, he translated Volume I of PIERRE-SIMON LAPLACE'S *Mécanique Céleste* into English. From 1806 to 1819 he ran a free *grammar school* in Nottingham. Afterwards, TOPLIS returned TO *Queen's College* as dean.

GREEN was accepted as a member of the *Nottingham Subscription Library*, a library that was financed by the annual subscription of its partners and whose purchases were determined by the wishes of the members. This also gave GREEN access to the *Transactions of the Royal Society of London* and he was able to find out about advances in mathematics and the natural sciences.

It is not known when GREEN had a relationship with JANE SMITH, the daughter of the manager of *GREEN 's Mill* – and, since his father was against such a union, their daughter named MARY ANN, born in 1824, was considered illegitimate and only registered under her mother's name. The lasting relationship between GEORGE GREEN and JANE SMITH resulted in a total of seven children; the last one was just a year old when GREEN died. The other children bore their father's family name, but the relationship was never legalised – not even after GEORGE's father died in 1829. There was a particular reason for this, which we will return to below. GREEN provided financially for his children and their mother. In the 1860s, however, the mill was no longer profitable and had to cease operations and the city seized the property.

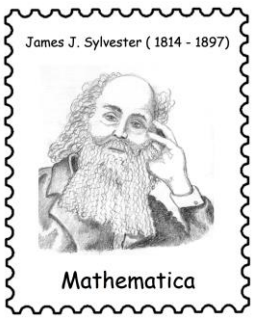
In 1828, 35-year-old GEORGE GREEN decided to have an article printed at his own expense. *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism* had 51 advanced orders – most of which came from members of the *Subscription Library*, none of whom probably really understood the content of the paper. Historically, it was the first attempt to describe electrical phenomena using the methods of analysis.

One of the subscribers was SIR EDWARD THOMAS FFRENCH BROMHEAD, a graduate of *Caius College*, Cambridge and the *Inner Temple*, London, and a member of the *Royal Societies* of London and Edinburgh. During his studies, together with JOHN HERSCHEL and CHARLES BABBAGE, he founded the *Analytical Society*, which set itself the task of translating the work *Sur le calcul différentiel et intégral* by SYLVESTRE FRANÇOIS DE LACROIX into English in order to help the more convenient LEIBNIZian notation achieve a breakthrough in Cambridge.



Even if BROMHEAD could not fully understand GREEN's explanations, he had an inkling of the importance of the work. He urged GREEN to write more articles and he covered their printing costs and ensured that they were published with the *Cambridge Philosophical Society* and the *Royal Society of Edinburgh*: *Mathematical Investigations concerning the Laws of Equilibrum of Fluids analogous to the Electric Fluid*; *On the Determination of the Exterior and Interior Attractions of Ellipsoid of Varying Densities*; *Researches on the Vibration of Pendulums in Fluid Media*.

Out of modesty, GREEN declined an invitation from BROMHEAD to visit his old university friends in Cambridge with him. However, he followed his advice to study at Cambridge to obtain an academic degree. Thanks to his inherited wealth, the now 40-year-old was financially able to stop working as a baker and miller and enrol at *Caius College*. The mathematical requirements did not give GREEN any trouble – but he had a lot of catching up to do in Latin, Greek and church history. In 1837 – contrary to the expectations of his much younger fellow students – he "only" graduated as fourth in his class (in the same year as JAMES JOSEPH SYLVESTER, who finished second best).



GREEN now hoped to get a permanent position at the university, but another two years passed before this happened, during which time he wrote further papers (*On the Motion of Waves in a Variable Canal of small Width and Depth*; *Note on the Motion of Waves in a Canal*; *On the Laws of Reflexion and Refraction of Light at the Common Surface of two non-crystallized Media*). It did not matter that he already had six children at this point – he fulfilled an essential requirement for his election as a *fellow*, namely that he was *not married*.

Tragically, his work as a *fellow* ended after just a few months. He fell seriously ill, returned to Nottingham and died there at the age of just 47, surrounded by his children and his beloved JANE.

During his lifetime there was almost no reaction to GREEN's writings and they were in danger of being forgotten. In 1845, four years after GREEN's death, the 21-year-old student WILLIAM THOMSON, later LORD KELVIN, recognised its importance and passed on his enthusiasm to JOSEPH LIOUVILLE and CHARLES FRANÇOIS STURM during a stay in Paris. Back in Cambridge he organised a new edition of the complete works (1850-1854). AUGUST CRELLE published GREEN's 1828 essay in his journal in the early 1850s.

Soon no one remembered Green in his hometown of Nottingham. As a miller, he “only” belonged to the class of craftsmen, and this was not an outstanding position in which one was valued. Added to this his “obscure” family circumstances were not held in high regard. His academic career took place in Cambridge and was not noticed in his home town.

It was only in 1937, after a letter of protest from the *Mathematical Section* of the *British Association for the Advancement of Science* to the Mayor of Nottingham, that GREEN's dilapidated grave was restored. The mill was also repaired in the 1970s and today there is a science centre.

Since there is no portrait of GEORGE GREEN, only a symbolic image of his mill remains as a reminder of the brilliant self-taught mathematician, about whom ALBERT EINSTEIN said in 1930 during a lecture at the University of Nottingham: *He was twenty years ahead of his time*.

GREEN's writings became the basis of theories developed in the second half of the 19th century by, among others, WILLIAM THOMSON (LORD KELVIN) and JAMES CLERK MAXWELL.

The mathematical methods used by GEORGE GREEN , which were developed – independently of him – a few years later by CARL FRIEDRICH GAUSS and GEORGE STOKES, cannot be presented in elementary terms within the scope of this biography.

What is important in the case of a plane curve is that *GREEN's theorem* can be used to calculate an area integral using a line integral. In the formulation of LEIBNIZ's so-called *sector formula,* this theorem reads:

If a smooth curve $\gamma(t): [a;b] \to \mathbb{R}^2$ *is given by a parametric representation, then a vector from the origin sweeps out an area whose oriented content can be calculated using an integral:*

$$A(\gamma) = \tfrac{1}{2} \cdot \int_a^b \left(x(t) \cdot y'(t) - y(t) \cdot x'(t) \right) dt$$

Examples:

The boundary of a circle can be described in an *x* - *y* coordinate system using the parametric representation $\gamma(t) = \left(\gamma_1(t) ; \gamma_2(t)\right) = \left(r \cdot \cos(t) ; r \cdot \sin(t)\right)$ for $0 \le t < 2\pi$.

Then the area *A* enclosed by the curve satisfies :

$$A = \tfrac{1}{2} \cdot r^2 \cdot \int_0^{2\pi} \left(\gamma_1(t) \cdot \gamma_2'(t) - \gamma_2(t) \cdot \gamma_1'(t) \right) dt$$

$$= \tfrac{1}{2} \cdot r^2 \cdot \int_0^{2\pi} \left(\cos(t) \cdot \cos(t) + \sin(t) \cdot \sin(t) \right) dt = \tfrac{1}{2} \cdot r^2 \cdot \int_0^{2\pi} 1 dt = \pi \cdot r^2.$$

For an astroid defined by $\gamma(t) = \left(\cos^3(t) ; \sin^3(t)\right)$, the result is analogous

$$A = \tfrac{1}{2} \cdot \int_0^{2\pi} \left(\cos^3(t) \cdot 3\sin^2(t)\cos(t) - \sin^3(t) \cdot (-3\cos^2(t)\sin(t)) \right) dt$$

$$= \tfrac{1}{2} \cdot \int_0^{2\pi} 3 \cdot \cos^2(t)\sin^2(t) \cdot \left(\cos^2(t) + \sin^2(t) \right) dt = \tfrac{1}{2} \cdot \int_0^{2\pi} 3 \cdot \left(\cos^2(t) \cdot \sin^2(t) \right) dt = \tfrac{3}{8} \pi.$$

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https://www.spektrum.de/wissen/george-green-war-mueller-baecker-und-nebenbei-mathematiker/2208430

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(Photo of GREEN's Mill: John O'Connor)