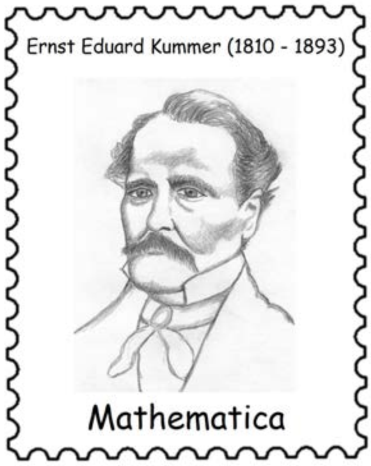
ERNST EDUARD KUMMER (January 29, 1810 – May 14, 1893)

by Heinz Klaus Strick, Germany



(drawings © Andreas Strick)

In 1813, when the remnants of Napoleon's "Great Army" flooded back after their Russian campaign, including the few survivors of the troop contingent of the Kingdom of Saxony, they brought deadly epidemics to their homelands. In Sorau, a small town in Lower Lusatia, the city physician Dr Carl Gotthelf Kummer worried about the sick and died himself from the consequences of a typhoid infection. His widow took on any work she could find to enable her two sons to receive an education. In 1828, the brothers Karl Volkmar and Ernst Eduard Kummer were even able to enrol for studies at the *Royal Friedrich University* in Halle-Wittenberg.

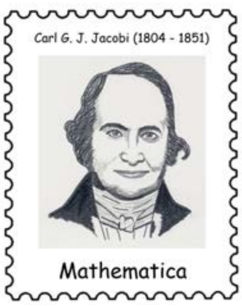
At the end of the first semester, Ernst Eduard informed his mother that he wanted to shift the focus of his studies from theology to mathematics – with the aim of becoming a teacher after graduation. In 1831 he submitted a treatise dealing with the series development of powers of the sine and cosine function. With this work he won prize money of 50 talers and – because of its exceptional quality – was even awarded a doctorate in philosophy. In the same month, he passed his exams as a teacher of theology, philology, German, history, mathematics and physics.

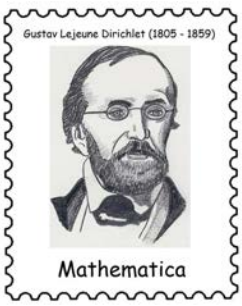
After a one-year traineeship at his former school in Sorau, he took up a position at the municipal grammar school in Liegnitz (Lower Silesia) in January 1833. However, his work as a teacher was soon interrupted by his being called up for military service as a musketeer in Breslau (now Wrocław). From there he sent the galley proofs of a treatise on special differential equations of the 2nd order to Carl Gustav Jacob Jacobi, professor in Königsberg, who reacted with surprise: "*Look, Prussian musketeers are now competing with professors with their mathematical work!*"

For ten years Ernst Eduard Kummer taught mathematics and physics in Liegnitz. His students included Ferdinand Joachimsthal and Leopold Kronecker. During this time he published several articles on analysis in Crelle's journal, and on Dirichlet's recommendation he was elected a corresponding member of the *Prussian Academy of Science* in Berlin.

Through Peter Gustav Lejeune Dirichlet he also met his later (first) wife Ottilie Mendelssohn, granddaughter of the philosopher Moses Mendelssohn and cousin of Dirichlet's wife Rebecka. In 1840 Kummer was appointed professor at the grammar school. As this did not involve any improvement in salary, he turned to his superior authority with an urgent request for a salary increase, as he would otherwise hardly be able to support his young family.

In 1842, Kummer completed his *habilitation* thesis on cubic residues, a prerequisite for being appointed to a post that had just become vacant at the University of Breslau. Kummer soon shifted the focus of his lectures from analysis to number theory.





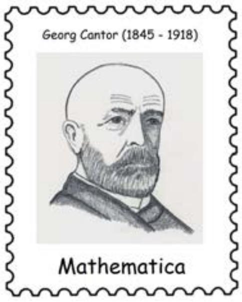
Among his first students was his former student LEOPOLD KRONECKER, who was able to complete his doctorate in 1845. In 1881, KRONECKER dedicated his contribution to KUMMER in the *Festschrift* for the 50th anniversary of his doctorate with the words *"... I owe you my mathematical existence*."

When GAUSS died in 1855, DIRICHLET was appointed to the chair in Göttingen. He proposed KUMMER as his successor in Berlin. To everyone's surprise, the latter in turn did not suggest either the generally favoured KARL WEIERSTRASS or his protégé KRONECKER for the chair in Breslau, but his former student JOACHIMSTHAL (who had been a full professor in Halle since 1853).

But soon it became clear what his aim was: within a year, KRONECKER *and* WEIERSTRASS were appointed to chairs in Berlin, which put Berlin, as a centre of mathematical research, in strong competition with Göttingen. However, the friendship and close cooperation of these three mathematicians came to an abrupt end in the mid-1870s after a fierce and irreconcilable dispute between KRONECKER and WEIERSTRASS.

KUMMER's obligations as successor to DIRICHLET included teaching at the *Berlin Military Academy* (*Kriegsschule*). In contrast to his predecessor, he particularly enjoyed this teaching position, where he had to deal intensively with questions of ballistics. In his opinion, the fact that these subjects required him to carry out experiments in order to gain knowledge only proved that the problems could not be solved with mathematical methods alone.

When he finished his 19 years of teaching at the military school in 1874, the relevant authorities examined whether he was entitled to a pension. However, KUMMER rejected such a pension, pointing out that over the years he had saved up his salary and that the interest on it was as much as the pension would amount to.



KUMMER's lectures were very popular because of his ability to present difficult ideas in a clear, structured, understandable and humorous way. Up to 250 students attended his lectures – an unusually large number for the time. During his time in Berlin he was able to supervise 39 doctorates, including GEORG CANTOR and HERMANN AMANDUS SCHWARZ.

For 15 years KUMMER held the office of secretary of the mathematical-physical department of the *Academy of Sciences* in Berlin and he was temporarily dean and rector of the university. In 1883, when he had the impression that his memory was deteriorating (nobody but he had noticed this), he decided to end his activities as a university teacher. His successor would be LAZARUS FUCHS, for whose *habilitation* KUMMER acted as reviewer.

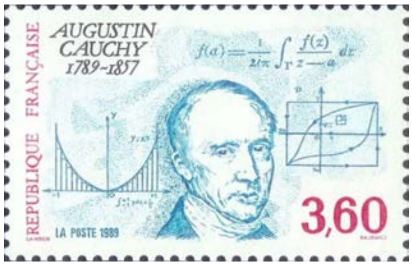
KUMMER's numerous contributions to the development of mathematics can be assigned to three creative periods: In the first years he worked intensively on problems from the theory of functions, in a second mainly on questions of number theory, and at the end of his research activities centred on problems of algebraic geometry – the figure on the right shows a so-called KUMMER surface. (source: Wikipedia)

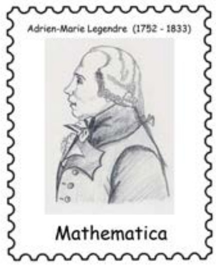
Since FERMAT's momentous marginal note on the margin of a DIOPHANTINE translation:

Cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet. (I have found a truly wonderful proof, but this margin is too narrow to grasp it.)

FERMAT's conjecture (usually abbreviated as FLT in the literature – *FERMAT's last theorem*) had occupied many important and less important mathematicians:

- The equation $x^n + y^n = z^n$ with $x, y, z \in IN$ has no solution for natural numbers $n > 2$.

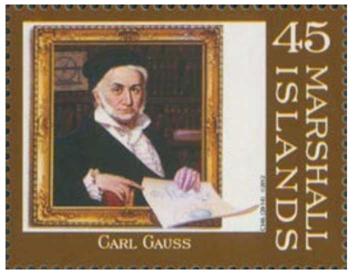




FERMAT himself had only been able to prove the case n = 4. EULER presented evidence for n = 3 in 1753 and DIRICHLET presented evidence for n = 5 to the *Académie des Sciences* in 1825, which was completed with the support of LEGENDRE.

In 1847 the French mathematician GABRIEL LAMÉ announced to the *Académie des Sciences* that he had found a proof for all prime numbers. AUGUSTIN-LOUIS CAUCHY reacted jealously to the announcement and claimed for his part that he was on the verge of completing the proof.

Both mathematicians suggested that they had succeeded in taking the decisive step of the proof. Through his regular correspondence with JOSEPH LIOUVILLE, KUMMER also learned of this, and knew immediately what mistake the two mathematicians had made and informed LIOUVILLE.

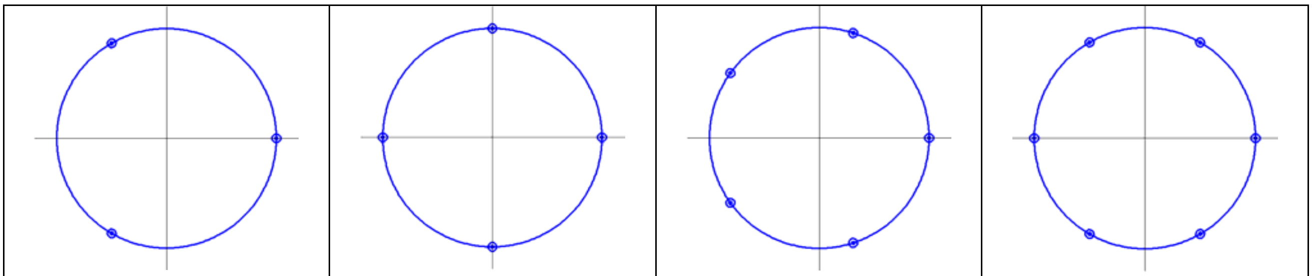


In the essential step in the proof both used the general validity of the fundamental theorem of arithmetic, a theorem which has been considered valid for natural numbers since EUCLID, but which was only completely proved by CARL FRIEDRICH GAUSS in his *Disquisitiones Arithmeticae* (1801). The theorem states that the decomposition of natural numbers into prime factors (except for the order of the factors) is unique.

For example, the number 6 can only be represented in the set as a product of the prime numbers 2 and 3. In the set $\mathrm{Z}[\sqrt{5}\cdot i] = \{a + b\cdot\sqrt{5}\, i \mid a, b \in \mathrm{Z}\}$ however, there is another decomposition, namely the representation as the product $(1+\sqrt{5}\cdot i)\cdot(1-\sqrt{5}\cdot i)$ and neither of these factors can be further decomposed themselves.

KUMMER had come across this phenomenon in 1844 while studying solutions of the *cyclotomic equations*, which are equations of the form $x^n = 1$.

According to the fundamental theorem of algebra proved by GAUSS, every nth-degree equation in the set of complex numbers has exactly n solutions. In GAUSS's complex plane, the solutions of $x^n = 1$ are points of the unit circle and vertices of regular n-gons with the coordinates $(\cos(\frac{2\pi\cdot k}{n}), \sin(\frac{2\pi\cdot k}{n}))$ with $0 \le k < n$.



Solutions of $x^3 = 1$ | Solutions of $x^4 = 1$ | Solutions of $x^5 = 1$ | Solutions of $x^6 = 1$

With the help of $\zeta = \cos\left(\frac{2\pi}{n}\right) + i \cdot \sin\left(\frac{2\pi}{n}\right)$, the *n*th root of the unit, the *n* solutions can also be written in the form $1, \zeta, \zeta^2, \ldots, \zeta^{n-1}$ where: $1 + \zeta + \zeta^2 + \ldots + \zeta^{n-1} = 0$ and $\zeta^n = 1$. For odd prime numbers *n*, the sum $x^n + y^n$ can then be written as the product
$(x+y) \cdot (x + \zeta \cdot y) \cdot (x + \zeta^2 \cdot y) \cdot \ldots \cdot (x + \zeta^{n-1} \cdot y)$.

If uniqueness of factorisation applied in the set of numbers of the form
$a_0 + a_1 \cdot \zeta + a_2 \cdot \zeta^2 + \ldots + a_{n-1} \cdot \zeta^{n-1}$ with $a_i \in \mathbb{Z}$, then the existence of further decompositions could be excluded. KUMMER discovered that this is not the case for *n* = 23, for example.

In order to be able to use the argument of unique factorisation, KUMMER introduced so-called *ideal numbers*. In the above example of the decomposition of the natural number 6, the *ideal numbers* $\sqrt{2}$, $\frac{1+\sqrt{5} \cdot i}{\sqrt{2}}$, $\frac{1-\sqrt{5} \cdot i}{\sqrt{2}}$ must be added to the set of numbers under consideration.

Then it is found that there are the following supposedly different product representations of the number 6:

$$6 = 2 \cdot 3 = \left(\sqrt{2} \cdot \sqrt{2}\right) \cdot \left(\frac{1+\sqrt{5} \cdot i}{\sqrt{2}} \cdot \frac{1-\sqrt{5} \cdot i}{\sqrt{2}}\right) = \left(\sqrt{2} \cdot \frac{1+\sqrt{5} \cdot i}{\sqrt{2}}\right) \cdot \left(\sqrt{2} \cdot \frac{1-\sqrt{5} \cdot i}{\sqrt{2}}\right) = \left(1 + \sqrt{5} \cdot i\right) \cdot \left(1 - \sqrt{5} \cdot i\right)$$

In 1850 KUMMER succeeded in showing the validity of the FLT for a very large number of exponents, namely for all *regular* prime numbers 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 41, 43, 47, 53, ... and their multiples. (A prime number *p* is called *regular* if *p* does not divide any of the numerators of the BERNOULLI numbers $B_2, B_4, B_6, \ldots, B_{p-3}$). The validity of the theorem remained open for the infinite number of *irregular* prime numbers 37, 59, 67, 101, 103, ...

After the *Académie des Sciences* had repeatedly offered a prize money in vain for the solution of the problem, in 1857 it awarded KUMMER – without having applied for it – the prize money in recognition of his merits. At that time, no one could have guessed that it would take another 138 years before the problem could be regarded as finally solved ... .

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https://www.spektrum.de/wissen/ernst-eduard-kummer-der-schoepfer-der-idealen-zahlen/1326924

Translated 2020 by John O’Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps.
Enquiries at europablocks@web.de with the note: "Mathstamps".