## JULIUS PLÜCKER (July 16, 1801 – May 22, 1868)

by HEINZ KLAUS STRICK, Germany

At the internet address www.genealogy.ams.org, a database has been set up which is designed in the form of an "academic family tree" showing who have written dissertations in the subject of mathematics (and in related fields such as mathematical education, computer science, statistics or theoretical physics).

On the opening page there is a graphic showing examples of the academic relationships between famous mathematicians such as EULER and HILBERT.



Kästmer Euler Pfaff Thibaut Laplace Lagrange Möbin Fourier Paisson Gubermann Gauß Dirksen Desele Weierstraß lacobi Dirichlet Gerling Phuc Gordan Lipschitz Kovalevskaya Klein Noether Lindemann Furtwängler Hilbert Taussley-Tood

And here – between CARL FRIEDRICH GAUSS and FELIX KLEIN – one can find the name of JULIUS PLÜCKER.

JULIUS was the eldest of three sons of a wealthy merchant from Elberfeld (today a district of Wuppertal). The boy attended a private school run by JOHANN FRIEDRICH WILBERG for children from the upper classes: a school where modern foreign languages were taught instead of Latin and Greek. Previously, WILBERG had been the head of the poorhouse, where he had taught children from the lower classes with great success. After moving to the *Privilegierten-Schule*, he could no longer personally look after the poorer children; but on Saturdays, the reforming educator, who was friends with ADOLPH DIESTERWEG, gathered teachers around him to talk to them about teaching, about contents and methods.

As there was no school in Elberfeld where the right to attend a university could be acquired, WILBERG recommended to JULIUS PLÜCKER's parents that he move to Düsseldorf to attend one of the oldest grammar schools in Germany, which – since the incorporation of the Rhine Province into Prussia – bore the name *Königlich Katholisches Gymnasium* (since 1945 it has been called: *Görres-Gymnasium*).

In 1819 JULIUS PLÜCKER enrolled at the University of Heidelberg for *Cameralia* (Public finance), a course of study designed to prepare him for work in the state administration. After three semesters he transferred to Bonn, where he attended lectures in physics, chemistry and mathematics and in 1823 he took his exams in Berlin.



PLÜCKER then travelled to Paris to improve his knowledge of geometry by attending the lectures of Augustin-Louis Cauchy, Sylvestre Lacroix and SIMÉON POISSON.



In the autumn he was awarded a doctorate (in absentia) by CHRISTIAN LUDWIG GERLING (University of Marburg) on the basis of a thesis on the application of methods of analysis in mechanics. In 1825, his application for habilitation was accepted by the University of Bonn.

From then on PLÜCKER took over lectures in analysis, algebra, geometry, accounting and astronomy as a private lecturer, partly also in French. In 1828 he was appointed Associate Professor. His main field of research was geometry and in 1829 and 1831 he published the 2-volume work Analytischgeometrische Entwicklungen (Analytical-geometric developments).

At the beginning of his approach to so-called *line geometry*, PLÜCKER pointed out that not only can points in the 2-dimensional coordinate system be unambiguously determined by the specification of two coordinates, but so can straight lines:

These can be described by their *line coordinates*, which result from the intersections of the lines with the coordinate axes.

A straight line that passes through the points (a, 0) and (0, b) has the equation  $\frac{x}{a} + \frac{y}{b} = 1$  in the "axis intercept" form with  $a, b \neq 0$ . From this the corresponding line coordinates  $(u, v) = (-\frac{1}{a}, -\frac{1}{b})$  can be read off. From the general equation Ax + By + C = 0 of a straight line, the line coordinates  $(u, v) = (\frac{A}{C}, \frac{B}{C})$  result accordingly.

A straight line can therefore also be described by the equation ux + vy + 1 = 0 where u, v each have a specific value and x, y stand for the coordinates of any points on the straight line. However, this equation can also be interpreted in such a way that it describes all straight lines that pass through a fixed point (x, y), i.e. a *pencil* of straight lines through this point.

So the equation ux + vy + 1 = 0 is also the equation of a point (x, y) in line coordinates. Because of this analogy between points and straight lines, propositions about points can be transferred to dual propositions about straight lines (and vice versa).

For example, the following two mutually dual theorems apply:

Three straight lines with the equations  $A_k x + B_k y + C_k = 0$  (k = 1, 2, 3) pass through a common

point if the condition:  $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0$  on the determinant of coefficients is satisfied.

Three points with the equations  $A_k x + B_k y + C_k = 0$  (k = 1, 2, 3) lie on the same straight line if

the condition:  $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0$  on the determinant is satisfied.

Analogously, in three-dimensional geometry, the equation Ax + By + C = 0 can be developed into the relationship ux + vy + wz + 1 = 0, which, on the one hand, describes the points of a plane, and, on the other hand, describes sets of planes that have a common point.

In 1833 PLÜCKER was appointed to Berlin – but only to a position as associate professor. Since such a position was "extraordinarily poorly paid" at the time, he was forced to teach mathematics at a grammar school at the same time. PLÜCKER was not happy with his position in Berlin, because, with regard to a vacant position as full professor, he has an important rival, JAKOB STEINER, whose *synthetic geometry* was in fierce competition with his *analytical geometry*.

In order to avoid a direct professional confrontation, he published his contributions on geometry in foreign journals. He received special recognition through feedback from ARTHUR CAYLEY and JAMES JOSEPH SYLVESTER.



When, at the end of the year, he was offered a position as a full professor in Halle, he accepted without hesitation, but remained there for only four semesters. In 1836 he returned to Bonn, where he married the following year and started a family.

For the next ten years he lectured mainly on geometry and in the meantime another work appeared: *System der analytischen Geometrie*, containing in particular a detailed theory of curves of the 3rd order. In this work he showed that one can distinguish 219 different types of curves of the third order. In 1839 he followed with *Theory of Algebraic Curves*.

Tired of the often polemic debate with the Berlin School around STEINER, PLÜCKER, who had always been interested in mathematical applications in physics, additionally took over the chair in physics in 1847. Inspired by the experiments of MICHAEL FARADAY, with whom he was in lively correspondence, he experimented with gas discharge tubes together with his doctoral student JOHANN WILHELM HITTORF, which ultimately led to the discovery of elementspecific line spectra and cathode rays. The success of the experiments was due

not least to the ingenious glassblower HEINRICH GEISSLER, whose achievements were recognised in 1868 when he was awarded an honorary doctorate by the University of Bonn.

In 1866 PLÜCKER received the *Copley Medal* of the *Royal Society* of London, which had already appointed him a member in 1855.

It is certainly no coincidence that immediately after the death of his competitor STEINER in 1863, PLÜCKER again devoted himself more strongly to geometrical questions. He was supported by his doctoral student FELIX KLEIN (whose dissertation topic was: On the transformation of the general equation of the second degree between line coordinates to a canonical form).

In 1868, the first part of the two-volume work *New Geometry of Space* was published, based on the consideration of the straight line as a spatial element.

PLÜCKER died before completing the second volume. The work was finally edited by KLEIN and published the following year.







First published 2018 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/julius-pluecker-von-der-geometrie-zu-elektrischenentladungen-im-vakuum/1607402

Translated 2021 by John O'Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".

