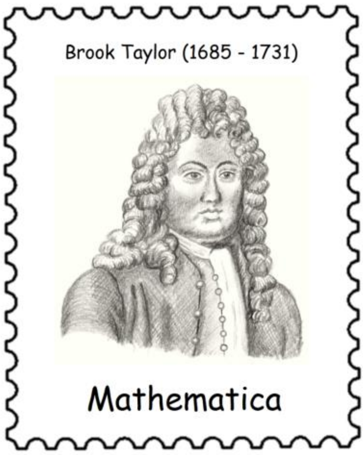
BROOK TAYLOR (August 18, 1685 – December 29, 1731)

by HEINZ KLAUS STRICK, Germany



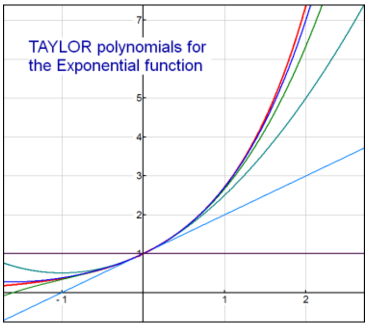
(drawing © Andreas Strick)

One of the most important theorems of differential calculus is named after the English mathematician BROOK TAYLOR.

The terms *TAYLOR's theorem* and *TAYLOR's series expansion* were first used in 1786 by the Swiss mathematician SIMON ANTOINE JEAN L'HUILIER, who also "invented" the *lim* notation after JOSEPH-LOUIS LAGRANGE had pointed out the importance of the theorem as fundamental for analysis in 1772 and had specified the statement of the theorem.

TAYLOR's text *Methodus Incrementorum Directa et Inversa* from 1715 contained, among other things, the statement that one can determine the function value of an (arbitrarily often differentiable) function at the point $x_0 + h$, i.e. in the vicinity of a point x_0, from the function value $f(x_0)$ and the values of the derivatives $f'(x_0)$, $f''(x_0)$, $f'''(x_0)$, ..., by, in today's notation: $f(x_0+h) = f(x_0) + h \cdot f'(x_0) + \frac{h^2}{2} \cdot f''(x_0) + \frac{h^3}{6} \cdot f'''(x_0) + \ldots$

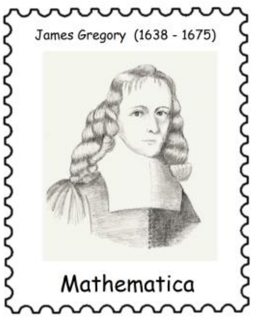
The writing contained – typically for the state of development of mathematics at the beginning of the 18th century – no consideration of the convergence of the series or of the estimation of the error. Also, from today's point of view, the published derivation seems awkward and partly erroneous, which, on the one hand has to do with NEWTON's fluxion notation, and on the other hand with TAYLOR's imprecise formulation and incomplete conclusions, but also with problems of the typesetter.



BROOK TAYLOR himself never claimed to be the "inventor" of this result. Similar versions had already been expressed by ISAAC NEWTON, GOTTFRIED WILHELM LEIBNIZ, JOHANN BERNOULLI and ABRAHAM DE MOIVRE – each independently of the other. The first example of a series development with the help of derivations comes from JAMES GREGORY (in the year 1671): $\theta = \tan(\theta) - \frac{1}{3} \cdot \tan^3(\theta) + \frac{1}{5} \cdot \tan^5(\theta) - \frac{1}{7} \cdot \tan^7(\theta) + \ldots$ and so $\arctan(x) = x - \frac{1}{3} \cdot x^3 + \frac{1}{5} \cdot x^5 - \frac{1}{7} \cdot x^7 + \ldots$

BROOK TAYLOR grew up in a wealthy home in Edmonton (near London). Until the age of 17, he was taught by tutors and his strict father insisted that art and music play a special role. In 1703 BROOK TAYLOR entered *St. John's College* in Cambridge.

During his studies, he met JOHN MACHIN, who was about the same age, who in 1706 discovered the relationship



$$\frac{1}{4} \cdot \pi = 4 \cdot \arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$

and, with the help of GREGORY's series expansion of the arc tangent, was able to calculate p to 100 decimal places.

The underlying series converges considerably faster than the series found by LEIBNIZ (independently of GREGORY):

$$\arctan(1) = \frac{1}{4} \cdot \pi = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots + \ldots$$

Although Taylor, at his father's request, aspired to a university degree as *Bachelor* and *Doctor of Laws* and achieved it in 1709 and 1714 respectively, mathematics played a more important role during his studies. In 1708 he wrote a paper on the oscillation centres of bodies, which was not published until 1714, however, and caused a fierce, lifelong priority dispute with Johann Bernoulli.

Already in 1712, on Machin's recommendation, he had been admitted as a member of the *Royal Society* and became a member of the commission that was to decide the priority dispute between Newton and Leibniz. However, Taylor – like the majority of the commission members – was too biased to be able to give an impartial judgement. From 1714 to 1718 Taylor even assumed the influential position of Secretary of the *Royal Society*, but then had to give up the post for health reasons.

In addition to the aforementioned theorem, *Methodus* contains a number of astonishing applications, including the investigation of the dependence of the frequency of a vibrating string on the tension of the string, its weight and its length, as well as the approach of finding approximate solutions of differential equations with the help of difference equations (*calculus of finite differences*).

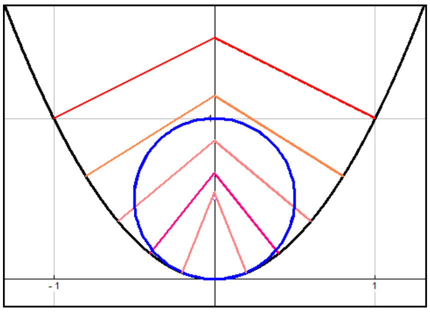
He explained how the derivative of an inverse function is related to the derivative of a function, and also derived the integration method of partial integration. Like Leibniz, who however did not publish this idea, he used the notation $y_1, y_2, y_3, \ldots$ for the 1st, 2nd, 3rd, ... derivatives of a variable y as well as y_{-1} for a variable whose derivative is y, i.e. for a primitive function of y.

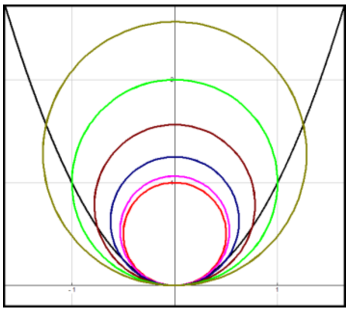
In addition to this work, *The Linear Perspective*: *or, a new method of representing justly all manner of objects as they appear to the eye in all situations* from 1715 is particularly noteworthy. Taylor was the first to give a formal definition of a vanishing point and to explain how the corresponding geometrical constructions are carried out. He also solved the inverse problem, i.e. how to determine the principal point and distance of the perspective as well as the true size of the side elevations from the perspective image of a cuboid.

This first mathematical treatise on the method of perspective representation was, however, very concise and difficult for artists to understand, and not only because of its definition-set-proof style. Taylor himself published an expanded version four years later, and after his death various editors tried to make it more readable. The work appeared in a total of 22 editions and was also translated into French and Italian.

Between 1712 and 1724, Taylor published 13 contributions on the mathematical theory of physical phenomena such as *capillarity* (the behaviour of liquids in narrow tubes) and *magnetic attraction*.

In a posthumously published contribution, Taylor showed that Newton's method for determining the curvature of a curve is equivalent to the method he developed. While Newton defined the centre of a circle of curvature as the limit of the intersection of normals to the curve, Taylor considered circles through three points on the curve which coincide in the limit case.





In another paper, he took up an idea of EDMOND HALLEY and developed a recursive method for determining zeros with the help of the first two derivatives:

Once one has found an approximate value for a zero that lies at the point $x_0 + h$, it follows from

$$0 = f(x_0 + h) \approx f(x_0) + h \cdot f'(x_0) + \tfrac{h^2}{2} \cdot f''(x_0),$$

that the following holds: $-f(x_0) \approx h \cdot \left(f'(x_0) + \tfrac{h}{2} \cdot f''(x_0) \right)$

and then $h \approx -\dfrac{f(x_0)}{f'(x_0) + \frac{h}{2} \cdot f''(x_0)} \approx -\dfrac{f(x_0)}{f'(x_0)}$,

and after substitution: $x_0 + h \approx x_0 - \dfrac{f(x_0)}{f'(x_0) - \dfrac{f(x_0) \cdot f''(x_0)}{2 \cdot f'(x_0)}}$.

Despite his unqualified support for NEWTON and the poisonous atmosphere between the English and Continental mathematicians, TAYLOR became friends with PIERRE RÉMOND DE MONTMORT and had a lively correspondence with him. He also mediated in the priority dispute between MONTMORT and DE MOIVRE and gave both of them suggestions for a joint solution of the *Knight's tour* problem.

BROOK TAYLOR only reached the age of 46. His unstable health was additionally affected by strokes of fate: His father did not agree with his choice of wife, as she comes from a good family but was not wealthy; he broke off all contact with him. After his wife died giving birth to their first child, and then the child died too, he was reconciled with his father. TAYLOR entered into a second marriage (this time his father agreed to the choice).

When the father died in 1729, he left his ailing son the travails of managing a large estate. The second marriage also ended in 1730 with the death of the wife during the birth of the first child though the daughter survived. Decades later, a grandson sifted through the unfinished writings left behind. They included a treatise on the calculation of logarithms, as well as philosophical and religious texts.

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https://www.spektrum.de/wissen/brook-taylor-englischer-mathematiker-pionier-der-differenzial-und-integralrechnung/1354083

Translated 2021 by John O’Connor, University of St Andrews

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