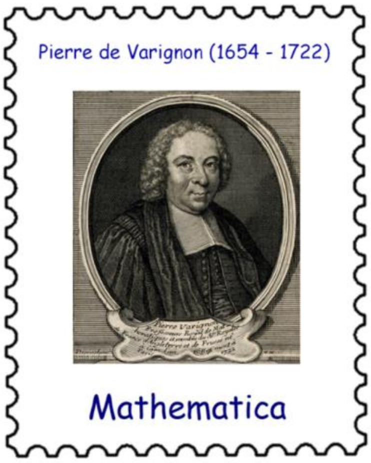
PIERRE DE VARIGNON (1654 – December 23, 1722)

by HEINZ KLAUS STRICK, Germany



It is an amazing fact that one of the most beautiful and simple theorems of geometry was not discovered by EUCLID (or any other mathematician of antiquity), but two thousand years later. The theorem can be found in the book *Élémens de Mathématique* by the French mathematician PIERRE DE VARIGNON, published in 1731, nine years after the author's death.

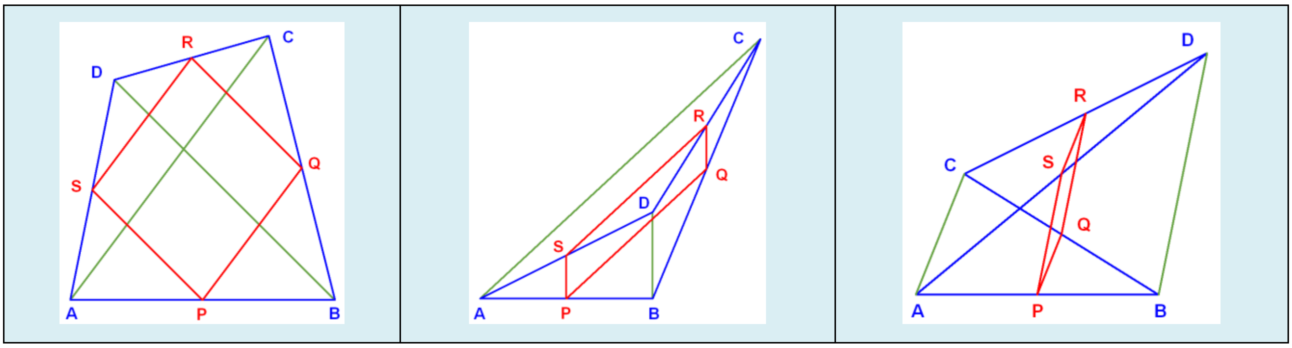
The editors write in the preface:

The principles of geometry are developed in this work with so much clarity and accuracy, the propositions are strung together in such a simple and natural way, the proofs are so short and simple that one will easily recognise in them the superiority of a genius who wrote this book.

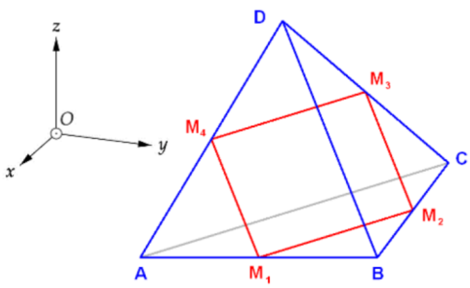
Theorem of VARIGNON:

If the centres of the adjacent sides of a quadrilateral are joined together, the result is a parallelogram whose area is half that of the quadrilateral and whose perimeter is equal to the sum of the lengths of the two diagonals.

This theorem is valid for any plane quadrilateral – for convex quadrilaterals where the diagonals are inside the quadrilateral, for concave quadrilaterals where one diagonal is inside and one outside the quadrilateral, and for overlapped quadrilaterals where both diagonals are outside the quadrilateral.



This theorem is also valid in the 3-dimensional (except for the area), i.e. for quadrilaterals whose vertices do not necessarily lie in a plane (tetrahedra). It is usually shown at the beginning of vector geometry lessons:



$$\overrightarrow{M_1M_2} = \tfrac{1}{2}(\overrightarrow{OB}+\overrightarrow{OC}) - \tfrac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB}) = \tfrac{1}{2}(\overrightarrow{OC}-\overrightarrow{OA}) = \tfrac{1}{2}\overrightarrow{AC},$$
$$\overrightarrow{M_4M_3} = \tfrac{1}{2}(\overrightarrow{OC}+\overrightarrow{OD}) - \tfrac{1}{2}(\overrightarrow{OD}+\overrightarrow{OA}) = \tfrac{1}{2}(\overrightarrow{OC}-\overrightarrow{OA}) = \tfrac{1}{2}\overrightarrow{AC},$$
$$\overrightarrow{M_1M_4} = \tfrac{1}{2}(\overrightarrow{OD}+\overrightarrow{OA}) - \tfrac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB}) = \tfrac{1}{2}(\overrightarrow{OD}-\overrightarrow{OB}) = \tfrac{1}{2}\overrightarrow{BD},$$
$$\overrightarrow{M_2M_3} = \tfrac{1}{2}(\overrightarrow{OC}+\overrightarrow{OD}) - \tfrac{1}{2}(\overrightarrow{OB}+\overrightarrow{OC}) = \tfrac{1}{2}(\overrightarrow{OD}-\overrightarrow{OB}) = \tfrac{1}{2}\overrightarrow{BD}.$$

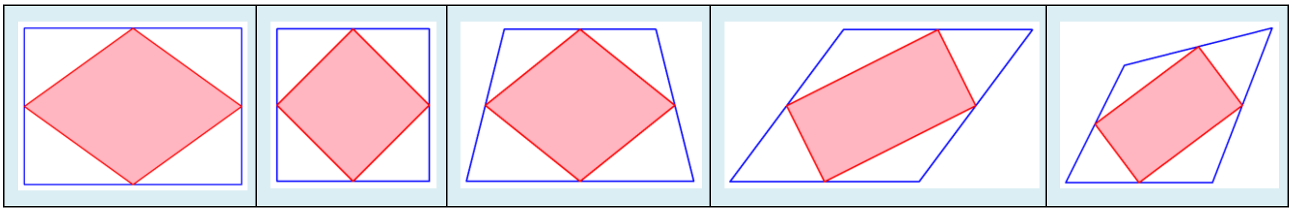
From all the figures it is clear that the parallelism of connecting lines of adjacent side centres to one of the diagonals results from the "Intercept Theorem" (EUCLID *Elements* VI.2).

From this also follow the statements about area and perimeter.

***Elements* VI.2** (Translation T. L. HEATH, 1908)**:**

If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally; and, if the sides of the triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle.

Furthermore, it is interesting to examine the special cases of VARIGNON's theorem:



PIERRE VARIGNON was born the son of a master builder in Caen (Normandy), where he (presumably) attended the Jesuit school and was prepared for the priesthood. At the age of 22 he took his vows and continued his studies, which he completed in 1682 with the Master's examination, and he was then ordained as a priest.

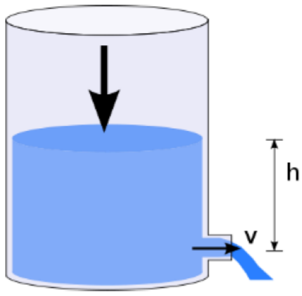
During his studies he became friends with CHARLES CASTEL, ABBÉ DE SAINT-PIERRE, later one of the most important philosophers of the *Enlightenment* in France, famous for his *Projet pour rendre la paix perpétuelle en Europe* (Plan for Perpetual Peace in Europe). CASTEL, who came from a wealthy family, did not depend on his income from the priesthood and he allowed VARIGNON to share in his financial independence.

CASTEL's special interest was mathematics. After studying the *Elements* of EUCLID and the geometry of RENÉ DESCARTES, a new world opened up for VARIGNON as well.

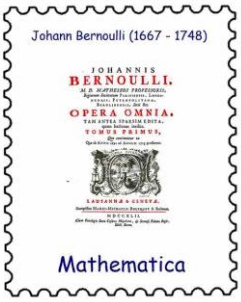
In 1686 the two went to Paris and made contact with other scientists there. In the following year VARIGNON published a first contribution (on pulleys with deflections) as well as the book *Projet d'une nouvelle méchanique*, in which he showed how the contributions of GOTTFRIED WILHELM LEIBNIZ on differential calculus, published in the previous years, could be applied to problems of mechanics. The book led to VARIGNON's admission to the department of *Géometrie* of the *Académie des Sciences* in 1688.

At the *Collège Mazarin* of the University of Paris, a post was created for him as professor of mathematics and he taught there for 34 years – until his death. In 1704, he was appointed professor at the *Collège Royal* (nominally a professor of Greek and Latin philosophy, but with no restrictions on the content of his teaching).

VARIGNON dealt with various applications of the differential calculus to problems of mechanics (parallelogram of forces, torque, equilibrium of liquids) and among other things he confirmed the law discovered experimentally by EVANGELISTA TORRICELLI in 1644, that the outflow velocity v of liquids depends only on the height h (the distance of the surface of the liquid from the opening) – in fact: $v = \sqrt{2gh}$.



He introduced the concept of instantaneous velocity and showed that the acceleration of a body could be obtained from this by differentiation. However, his individual contributions were lost in the abundance of publications.



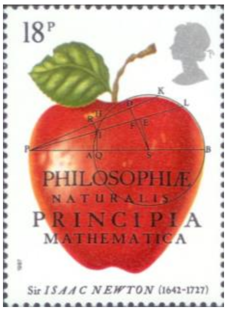
In 1692 VARIGNON met the Basel-born mathematician JOHANN BERNOULLI, and a lifelong friendship developed between the two. In 1696 GUILLAUME DE L'HOSPITAL's *Analyse des infiniment petits* was published, the first book on LEIBNIZ's differential calculus, which was essentially based on material by JOHANN BERNOULLI. Among the critics of the work was MICHEL ROLLE, who was committed to mathematical tradition and exactness, which in his opinion were lost by considering infinitely small quantities. VARIGNON, on the other hand, defended working with infinitesimal quantities, but also warned against too careless a handling.

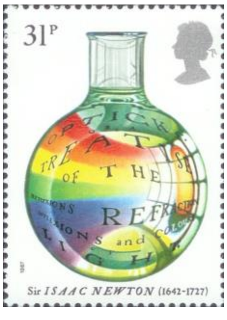
After repeated heated arguments between the traditionalists and the innovators at the meetings of the *Académie*, the committee decided not to put the subject on the agenda any more.

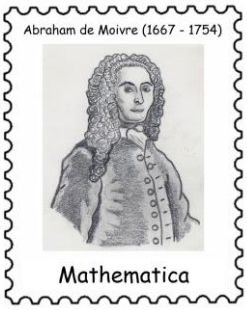
In 1712 VARIGNON was appointed director of the *Académie des Sciences*. Despite his good relations with both LEIBNIZ and ISAAC NEWTON, he did not succeed in mediating in the priority dispute. His position between the two camps was respected and recognised by both sides.

When the second edition of the *Principia* appeared in 1713, NEWTON sent him one of his six copies. Of the second edition of the *Opticks*, NEWTON sent three copies to the *Académie*, and VARIGNON forwarded one copy to JOHN BERNOULLI, hoping to build a bridge through this gesture.

In his letter of thanks to NEWTON, VARIGNON expressed his deep regret that the continuing disputes between NEWTON's and LEIBNIZ's camps were preventing a meaningful exchange.







When in 1722 the correspondence between BERNOULLI and NEWTON once again degenerated, VARIGNON intervened and tried to mediate together with ABRAHAM DE MOIVRE, who lived in exile in London.

VARIGNON's contributions and his role as mediator between the camps were also recognised by his admission to the *Prussian Academy of Sciences* (1713) and the *Royal Society* (1718).

His extensive teaching activities, his assumption of the responsible position as head of the *Académie* and the extensive correspondence associated with it prevented VARIGNON from elaborating his own ideas.

After his death, former students compiled his teaching notes into the teaching work *Élémens de mathématiques*.

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https://www.spektrum.de/wissen/pierre-de-varignon-der-vermittler-zwischen-newton-und-leibniz/2056071

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Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".