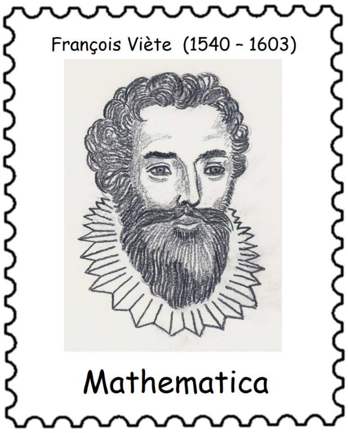
FRANÇOIS VIÈTE (1540 – December 13, 1603)

by HEINZ KLAUS STRICK, Germany



(drawings © Andreas Strick)

When he was born, no one could have suspected that FRANÇOIS VIÈTE, son of a wealthy lawyer in Fontenay-le-Comte (Poitou, today Vendée) would become one of the most important mathematicians of the 16th century. The first years of his life were keeping with his status: he attended the local Franciscan monastery school, studied law at the University of Poitiers, took his exams there at the age of 20 and successfully started working as a lawyer – with the clear aim of pursuing a university career after a few years of professional experience.

In 1564, however, the influential family of JEAN DE PARTHENAY, the military leader of the Huguenots, offered him a job as secretary. Here he was not only responsible for legal issues, but also for the upbringing and education of the 11-year-old daughter CATHERINE. For her he wrote his own texts on scientific and mathematical topics, especially astronomy, trigonometry and geography, some of which are still preserved today. He was already using decimal numbers 20 years before SIMON STEVIN published *De Thiende*.

When JEAN DE PARTHENAY died, VIÈTE accompanied his widow and daughter to La Rochelle, where he met the leaders of the Huguenot aristocracy, including HENRI DE NAVARRE, who later became King HENRI IV.

From 1570 he lived again in Fontenay-le-Comte and he also enrolled as a lawyer in Paris. In August 1572 he was in Paris on the occasion of HENRI DE NAVARRE's wedding to MARGARETE DE VALOIS, daughter of Queen CATHERINE DE MEDICI. This wedding was supposed to end the decades of violent clashes between the Catholic and Protestant camps, but on the night of August 23rd to 24th, there was a massacre: the leader of the Huguenots and many of his followers were killed (*St Bartholomew's Day massacre*).

For the next eight years VIÈTE worked as a lawyer in Rennes, the capital of Brittany, away from Paris, and became a member of the regional court. During this time he also brokered the marriage of his former student CATHERINE to the Protestant duke RENÉ DE ROHAN, who came from one of the most influential noble families in Brittany. On his recommendation, King HENRI III entrusted VIÈTE with the important office of *Maître des Requêtes* whose duty was to decide on incoming requests for the Supreme Court in Paris.

VIÈTE continued to work as a lawyer and when he won a case for RENÉ DE ROHAN's sister against the (Catholic) DUKE OF NEMOURS, the Catholic *Holy League*'s patience was over and they put pressure on the king until he dismissed FRANÇOIS VIÈTE from office for allegedly preferring Protestants.

And so in 1585 VIÈTE withdrew for four years, first to Fontenay, then to Beauvoir-sur-Mer, in order to devote himself entirely to mathematics ...

HENRI III, who tried to ensure peaceful coexistence between Protestants and Catholics, came under increasing pressure from the *Holy League*. When the childless ruler tried to nominate the Huguenot HENRI DE NAVARRE as his successor, the political situation worsened dramatically.
The *Holy League* allied with PHILIP II of Spain, who sent troops into the country and after the murder of HENRI III even claimed the French throne for his daughter ISABELLA. An encrypted letter from PHILIP to his allies, containing the precise military plans of the Spanish troops, was intercepted and decoded by FRANÇOIS VIÈTE, who was at the side of the new King HENRI IV.

When PHILIP II found out about this, he accused the French king of black magic, because his advisors had assured him that the encryption, in which over 500 characters were used, could not be broken. A complaint to the Pope was rejected; after all, mathematicians in the Vatican had for some time been able to read the correspondence of the Spanish king, which was only encrypted with mono-alphabetic substitution.

It was only when HENRI IV converted to the Catholic faith ("*Paris was worth a mass*") that he could finally enforce his claim to rule in France and also establish peace in the country for a few years (*Edict of Nantes*). FRANÇOIS VIÈTE served the new king as a personal advisor until 1602. Then, exhausted, he withdrew to his estates and a year later he died in Paris.

FRANÇOIS VIÈTE never called himself a mathematician ("*ego qui me Mathematicum non profiteor* ..."). Since his activity as teacher of CATHERINE DE PARTHENAY, however, he has been fascinated by mathematical questions. While he was working as a lawyer in Paris, he was busy with mathematics in every free minute.

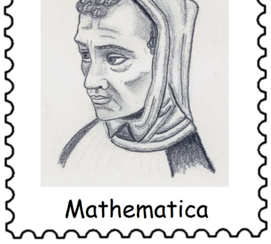
In 1571 he published his first book: *Canon mathematicus* – a compilation of all basic trigonometric problems for plane and spherical triangles, including a table of the six trigonometric functions.

In 1591 his main work *In artem analyticem isagoge* was published under his Latinized name FRANCISCUS VIETA. It was an introduction to algebra. He dedicated the book to his pupil CATHERINE, to whom he – as he emphasised – owed his love for mathematics.

Although there had been many approaches to algebraic notation in the previous decades, none of his predecessors dealt with it as consistently as he did. VIETA used consonants as placeholders for constant magnitudes, and vowels for unknown magnitudes (the lowercase letters *x, y, z* commonly used today were introduced by DESCARTES in 1637). Plus and minus signs were used for addition and subtraction, and a fraction line for division. However, he still wrote the word *in* for multiplication and *aequale* instead of the equals sign. Powers of variables were given by adding words such as *quad.* (*quadratus*), *cubus* and so on.

In the first chapter of the book he made it clear that it was necessary to give a desired quantity a name and to introduce a symbol for it. In the second chapter he explained the basic laws of computation (for example *associative* and *distributive laws*) as well as permissible transformation steps for equations, which – as he noted – do not change equality (what is now known as an *equivalence*). Proportions (equations in the form of ratios) also played a special role; for equations that can be represented in the form of a proportion led directly to geometric solutions (for three given members of a proportion, the fourth could be determined by construction).

In the third chapter he stated that only quantities of the same dimension could be compared with one another, i.e. lengths, areas, volumes, but also quantities of higher dimensions. This was expressed in the equations he dealt with by providing the coefficients with corresponding exponents, for example in the case of an equation of the 3rd degree its normal form was written: $A^3 + BA^2 + D^2A = G^3$.



In the following chapters, VIETA demonstrated how to solve equations.

In the collection of problems *Zeteticorum libri quinque*, which appeared only two years later, he succeeded in demonstrating the advantages of his algebraic method in an impressive way.

The following example – like most of the tasks he treated – was taken from the *Arithmetica* of the DIOPHANTUS.

Among other things, DIOPHANTUS had set the task:

- *To divide a given number into two parts, the difference between which was given*.

  Let the given number be designated *D* and the difference between the two numbers searched for by *B*, and the smaller of the two numbers by *A*. Then the larger one equals $E = A + B$.

  The equation $A + (A + B) = D$ can be converted to $2A + B = D$ or $2A = D - B$ and then $A = \frac{1}{2}D - \frac{1}{2}B$.

  For the larger number *E* therefore it follows: $E = A + B = \frac{1}{2}D - \frac{1}{2}B + B = \frac{1}{2}D + \frac{1}{2}B$.

  So you find the two numbers *A* and *E* by taking half the sum and half difference respectively of the quantities *D* and *B*.

In the second chapter of the book he treated quadratic equations and among other things VIETA solved the following problem of DIOPHANTUS by reducing it to the task solved above:

- *Two numbers are to be found when the rectangle of the numbers* (i.e. the product) *and the sum of the numbers are given.*

  First he shows in general that the following holds: $(X + Y)^2 - 4XY = (X - Y)^2$ and then concludes that one can calculate the square of the difference $X - Y$ of the two numbers from the square of the sum $X + Y$ of the numbers by subtracting the fourfold product $4XY$.

  Then if you know the sum and the difference of the two numbers, you can determine the two numbers you are looking for (as outlined above).

In the publications *De recognitione aequationum* (Investigation of equations) and *De emendatione aequationum* (Improvement of equations), published after his death, VIETA showed how one can get to other types of equations by transforming certain equations and, conversely, how one can use these modifications to solve problems.

This was how he showed the set of rules (later named after him):

- If a quadratic equation in *A* has the form $(B + D) \cdot A - A^2 = BD$ then *B* and *D* are the two solutions.
- If a cubic equation in *A* can be written as $A^3 - (B + D + G) \cdot A^2 + (BD + BG + DG) \cdot A = BDG$ then *B*, *D*, *G* are the solutions.

He also demonstrated how equations could be simplified by substitution.

For example, by the substitution of $E - B$ for *A* the cubic equation $A^3 + 3BA^2 = D^3$ can be converted to $E^3 - 3B^2E = D^3 - 2B^3$.

He noted that in principle, in an equation of *n*-th degree the term of (*n*-1)-th degree can be eliminated.

Obviously VIETA knew how to deal with binomial formulas, even if he wrote them (compared to our notation today) in a complicated way:

$(A + B)^5$ is *A quadrato-cubus*, + *A quadrato-quadrato in B* 5, + *A cubo in B quadratum* 10, + *A quadrato in B cubum* 10, + *A* in *B quadrato-quadratum* 5, + *B quadrato-cubo*.

He also noticed a similar regularity in the addition theorems for sine and cosine.

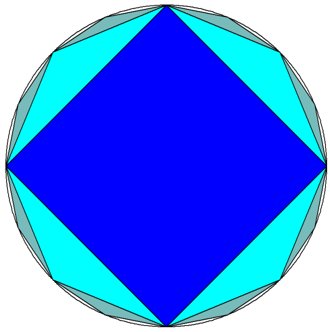
For example, he recognised, 130 years before DE MOIVRE:

$$\cos(5\alpha) = \cos^5(\alpha) - 10\cos^3(\alpha)\sin^2(\alpha) + 5\cos(\alpha)\sin^4(\alpha)$$

$$\sin(5\alpha) = 5\cos^4(\alpha)\sin(\alpha) - 10\cos^2(\alpha)\sin^3(\alpha) + \sin^5(\alpha)$$

This knowledge of trigonometric functions helped VIÈTE to solve a problem that the Dutchman ADRIAAN VAN ROOMEN posed to the mathematicians of Europe. VIÈTE realised that to do this, a 45-degree equation must be solved. He succeeded in doing this in the shortest possible time using a trigonometric approach. "*Ut legi, ut solvi*" (as read, so solved) he commented on his success.

Other writings followed at short intervals, some of which were only published posthumously. His method for determining the number π was particularly impressive.



It repeatedly compared the ratios of the area of the regular $2n$-gon to that of the $2n+1$-gon and thus reached an infinite product – the first product representation of the circle number π in the history of mathematics:

$$\frac{2}{\pi} = \frac{A_4}{A_8} \cdot \frac{A_8}{A_{16}} \cdot \frac{A_{16}}{A_{32}} \cdot \ldots = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \ldots$$

The strain of being the king's personal advisor and the ever-increasing preoccupation with mathematical problems contributed to VIÈTE's exhaustion. In the last years of his life, he wasted additional energy by arguing fiercely with CHRISTOPHER CLAVIUS, the advisor to Pope GREGORY XIII, accusing him – falsely – of having carried out the calendar reform on the basis of incorrect calculations and arbitrary settings.

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https://www.spektrum.de/wissen/francois-viete-im-nebenberuf-schoepfer-der-modernen-algebra/1214470

Translated 2020 by John O’Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps.
Enquiries at europablocks@web.de with the note: "Mathstamps".