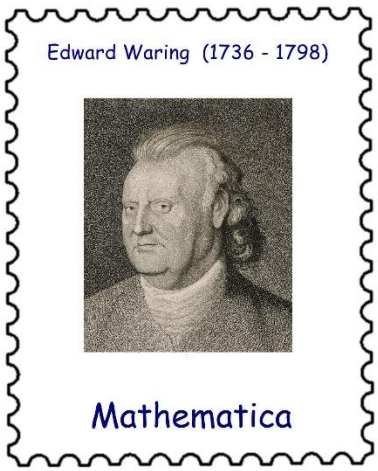
EDWARD WARING (1736 – August 15, 1798)

by HEINZ KLAUS STRICK, Germany



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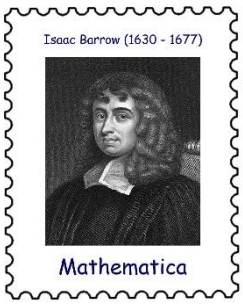
When in 1796 the French mathematician JÉRÔME LALANDE remarked in an article that there was not a single first-class analyst in the whole of England, the English mathematician EDWARD WARING was very indignant and pointed out in a statement that in his *Meditationes Algebraicae* (published in 1762) he alone had formulated *between 300 and 400 theorems of one kind or another* and that his work had been highly praised by mathematicians on the continent, such as LEONHARD EULER and JEAN LE ROND D'ALEMBERT. JOSEPH-LOUIS LAGRANGE had even described the *Meditationes* as *a work full of excellent research*.

When WARING's statement was finally published (in The *Monthly Magazine*), WARING had already died.

It is not known exactly when EDWARD WARING was born; based on various sources, the year 1736 is assumed to be the most likely year of his birth. His parents, JOHN and ELIZABETH WARING, ran a farm in Old Heath, a village near Shrewsbury (Shropshire, West Midlands), where EDWARD also attended school. In 1753 he was accepted to study at Magdalene College, Cambridge; he partly earned his living by working as a *sizar* (servant for other students). From the outset he impressed his teachers with his mathematical abilities. In 1757 he graduated as senior *wrangler* with a Bachelor of Arts (BA) degree. The following year he was appointed a fellow of his college.

WARING planned to publish a comprehensive work entitled *Miscellanae Analytica*. With the first chapter completed, he applied for the position of *Lucasian professor* at Trinity College, Cambridge University. The chair, founded in 1663 by HENRY LUCAS, a member of the English House of Commons, is still considered one of the most prestigious in the world. Initially, the chair was held by ISAAC BARROW (1663-1669) and his student ISAAC NEWTON (1669-1702). Later, CHARLES BABBAGE (1828-1839), GEORGE GABRIEL STOKES (1849-1903), PAUL DIRAC (1932-1969), and STEPHEN HAWKING (1979-2009), among others.



Although WARING was only 23 years old and lacked *a master's degree*, he was appointed *Lucasian Professor* in early *1760* —despite the polemical, anonymous objections of WILLIAM POWELL, one of the teachers at St John's College. WARING held the professorship for 38 years (until his death). Before his appointment, he was granted the missing title of MA (*Master of Arts*) by royal decree.

WARING published his completed work, *Miscellanea Analytica*, which dealt primarily with problems in number theory and the solution of algebraic equations. The text was even translated into Italian by VINCENZO RICCATI in 1770.

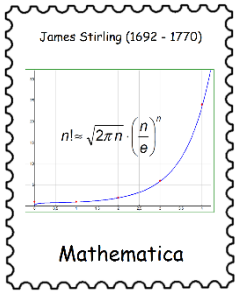
WARING was admitted to the *Royal Society*. In 1767, he received his MD (Doctor of Medicine) degree and, for a time, actually practised as a physician; however, he was soon forced to give this up due to his severe myopia.

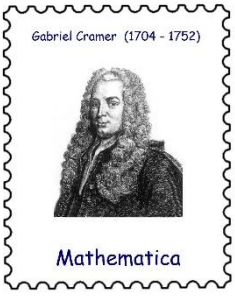
WARING wrote numerous articles that were printed in the *Philosophical Transactions* of the *Royal Society.* His main work*, Meditationes Algebraicae*, was published in 1770. In it he expanded on the investigations dealt with in *Miscellanea Analytica* and also addressed the convergence of sequences and series as well as the geometry of conic sections. It is clear from these remarks that he was one of the few English mathematicians of his time who was up to date with the latest mathematical contributions published on the continent. In his investigations into the theory of equations, he dealt, among other things, with circle division equations $x^n - 1 = 0$ and the relationship between the coefficients and the roots of a polynomial (the so-called resolvent equations), which can be seen as a precursor to the development of what is known as GALOIS theory.

WARING was the first mathematician in Great Britain to consider partial flux equations (in NEWTONIAN notation) – analogous to partial differential equations in LEIBNIZ's notation. Like all his works, these explanations suffered from the fact that he sometimes used one notation and sometimes another. In general, some of his formulations were difficult to understand, and the structure of his texts was not always systematic.

As a Lucasian professor, he was not required to give lectures; he may therefore have lacked immediate feedback on the comprehensibility of his presentations.

WARING published the work *Proprietates Algebraicarum Curvarum*, in which he classified, among other things, curves of the 4th order and generalized some of the results of ISAAC NEWTON, JAMES STIRLING, LEONHARD EULER and GABRIEL CRAMER.





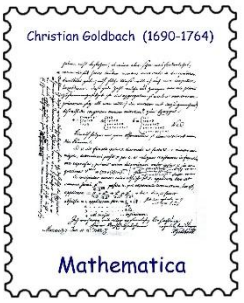
The work also included a criterion usually attributed to the French mathematician JEAN LE ROND D'ALEMBERT (published in 1768):

Quotient rule for infinite series

If for a sequence of numbers $(a_n)_{n\in\mathbb{N}}$ there exists a number $q<1$ such that for almost all $n \in \mathbb{N}$, $\left|\frac{a_{n+1}}{a_n}\right| \le q<1$ then the series belonging to the sequence $(\sum_{k=0}^{n} a_k)_{n\in\mathbb{N}}$ is absolutely convergent.

Example: For $a_n = \frac{x^n}{n!}$ the following applies: $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right| = \left|\frac{x}{n+1}\right|$, and for each *x* a suitable number $q<1$ can be specified so that the above condition is fulfilled.

In the *Meditationes,* WARING also published two sentences that had not previously appeared in a printed work:



- Every even number can be represented as the sum of two prime numbers.

(the so-called GOLDBACH conjecture, which CHRISTIAN GOLDBACH formulated in 1742 in a letter to LEONHARD EULER)

- For every prime number *p,* the number $(p-1)!+1$ is divisible by *p* .

This theorem actually originates from WARING's student JOHN WILSON , who graduated top of his class in the Mathematical Tripos at Cambridge four years after WARING, but then pursued a career as a lawyer. This statement, known in the specialist literature as *WILSON 's theorem*, was proved in 1773 by JOSEPH-LOUIS LAGRANGE, who also proved the converse of the theorem, namely the statement:

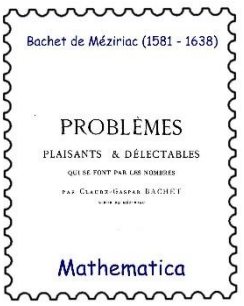
- *If a natural number n divides the number* $(n-1)!+1$*, then n is a prime number*.

The work also contains a problem that is still associated with the name of the English mathematician today – it concerns the property that all natural numbers can be represented as the sum of powers with a fixed exponent:

WARING's conjecture

For every natural exponent $k>1$ there *exists* a natural number $g(k)$ with the following property: Every natural number is representable as the sum of at most $g(k)$ k-th powers of natural numbers.

WARING was inspired by the so-called *4-square theorem,* which CLAUDE GASPARD BACHET DE MÉZIRIAC had conjectured in 1621 and LAGRANGE had proved in 1770:



- Every natural number can be represented as the sum of a maximum of four square numbers, i.e.: $g(2)=4$.

Analogously, WARING investigated the representation of natural numbers as the sum of third and fourth powers; for this he put forward the following conjectures:

- Every natural number can be represented as the sum of a maximum of nine cubes, i.e.: $g(3)=9$.
- Every natural number can be represented as the sum of a maximum of nineteen fourth powers, i.e.: $g(4)=19$.

The proof of the *existence* of such maximal numbers *g* (*k*) was not provided until 1909 by DAVID HILBERT.

However, the concrete determination of *g* (*k*) proved to be complicated.

EDMUND LANDAU, for example, was only able to prove in 1909 that there is only a finite number of natural numbers for which the maximum number of nine cubes is required.

Eventually, it turned out that only for the numbers 23 and 239 are nine cubes actually necessary: $23 = 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 2^3 + 2^3$; $239 = 1^3 + 2^3 + 2^3 + 2^3 + 2^3 + 3^3 + 3^3 + 3^3 + 5^3$. For numbers larger than 1.2 million, five cubes are always sufficient.

The proof for $g(4) = 19$ was not achieved until 1986.

Finally, for *k* = 5, In 1964, CHEN JINGRUN showed that $g(5) = 37$.

In 1784, WARING was awarded the COPLEY Medal by the Royal Society, Britain's highest award for scientific achievement. The Academies of Sciences of Göttingen and Bologna honoured him by admitting him to membership.

In his final years, he fell into a deep religious melancholy. He never recovered from a severe cold; he died at the age of about 62.

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