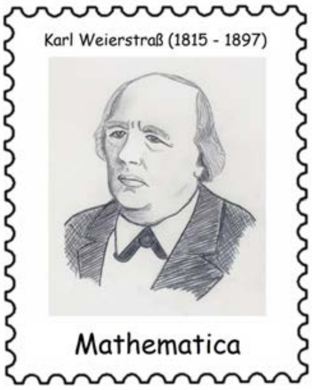
KARL WEIERSTRASS (October 31, 1815–February 19, 1897)

by Heinz Klaus Strick, Germany



(drawing © Andreas Strick)

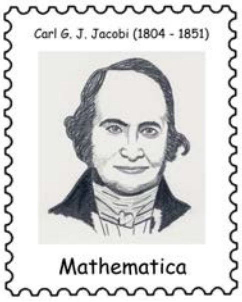
KARL THEODOR WILHELM WEIERSTRASS is frequently referred to in the mathematical literature as the "father of modern analysis". Yet to this day, not a single postage stamp has been issued to honour this important German mathematician.

KARL WEIERSTRASS's father was employed for a time as secretary to the mayor of the municipality of Ostenfelde, in the Münster region. Later, he worked as a revenue inspector. Because the father was now a member of the Prussian civil service, the family moved frequently.

An outstanding high-school student, KARL took an interest in mathematics; he regularly read the papers in the specialist *CRELLE's Journal.* KARL finished high school in Paderborn, and at his father's wish, undertook studies in law and finance at the University of Bonn.

However, he had no real interest in these subjects. He was torn between his desire to comply with his father's wishes for his future career and his genuine desire to study mathematics. Unable to reconcile this conflict, he neglected the lectures in his subject areas, instead reading intensely the works of LAPLACE, ABEL and JACOBI, though he did not participate in any of the functions of the mathematical faculty. Indeed, he spent most of his time in the rooms of a student fraternity.

During his eighth semester, he finally decided to break off his studies without sitting for the final examinations. A friend of the family convinced the outraged father to permit his son to attend the teachers' academy in Münster so that he could prepare for a career as a high-school teacher of mathematics and physics. Here, KARL WEIERSTRASS encountered CHRISTOPH GUDERMANN as one of his teachers. The latter had completed his doctorate under the direction of CARL FRIEDRICH GAUSS, and his first lecture was on elliptic functions, a topic that at the end of the 1820s had become a central object of mathematical research through the work of NIELS HENRIK ABEL and CARL GUSTAV JACOB JACOBI.



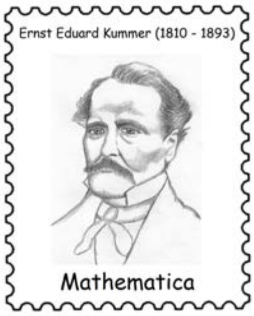
WEIERSTRASS very quickly completed the required course of study, taught in Münster during a one-year trial period, and in 1842, assumed a teaching position at a school in West Prussia. In the meantime, he had completed several of his own articles on elliptic functions, which GUDERMANN later judged to be on a par with the work of ABEL und JACOBI.

However, as a generalist teacher (he also had to teach German, history, botany, calligraphy, and gymnastics), he was unhappy to be cut off from the scientific literature and current journals. He published his own results on elliptic functions in his school's annual report, where (as was to be expected) they were noticed by no one. His health suffered from this unsatisfactory situation, and he increasingly was beset by dizzy spells.

It was only when his paper *Zur Theorie der ABELschen Funktionen* (on the theory of abelian functions) appeared in 1854 in *CRELLE's Journal* that his unhappy situation altered, and it did so in a dramatic way.

Within of few weeks of the publication of his article, the University of Königsberg awarded him an honorary doctorate, and he was granted a year's leave from teaching duties, through payment of his salary, so that he could continue his research. His application for a professorship at the University of Breslau was, unfortunately, unsuccessful, though only for the reason that ERNST EDUARD KUMMER, who had given up his professorship in Breslau to accept a chair in Berlin, was intent on bringing WEIERSTRASS to Berlin with him.

In the summer of 1856, WEIERSTRASS took up his duties as lecturer at the Berlin Industrial Institute (today the *Technical University*). That October, he was offered a position at the University of Berlin, though he could assume the post only later because of his obligations to the Berlin Industrial Institute.



The collaboration between his Berlin colleagues ERNST EDUARD KUMMER and LEOPOLD KRONECKER and his own novel lectures on physical applications of FOURIER series, complex analysis, and elliptic functions turned the University of Berlin into a new centre of mathematics, to which students flocked from all over the world.

WEIERSTRASS's lectures *Introduction to Mathematical Analysis and Integral Calculus* from the years 1859 to 1861 represent the beginning of a coming to grips by mathematicians with the foundations of analysis. Whereas previously, most theorems had been established only intuitively, WEIERSTRASS sought, with what came to be called WEIERSTRASSian rigor, precise argumentation that rested on formally established results.

One may say that the style and structure of WEIERSTRASS's lectures for beginning students has survived to this day: the construction of the real numbers, approximation of functions by power series, the ϵ–δ definition of continuity, differentiability, analytic continuation from the real numbers $\mathbb{R}$ to to the complex numbers $\mathbb{C}$, singularities (points in $\mathbb{R}$ or $\mathbb{C}$ at which a function is undefined), analytic functions of several variables, line integrals.

At the end of 1861, WEIERSTRASS suffered a physical collapse. He required a year's leave before he could continue his work as a university professor. Even then, he had to sit at the lectern and dictate to a student who wrote at the blackboard.

Among the large number of WEIERSTRASS's outstanding students, there is one name that deserves special mention: SOFIA KOWALEWSKAJA.

She came to Germany in 1870 because women were not allowed to study at Russian universities. But in Berlin as well, despite WEIERSTRASS's intercession, women were not even allowed to be present at lectures.

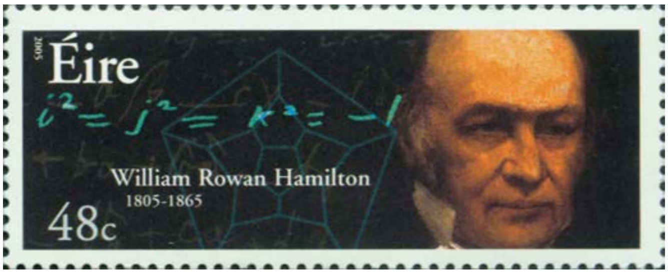
Therefore, he tutored her privately twice a week and supervised her first publications. After a considerable search, he finally found a university that would grant her a doctoral degree on the basis of her high-quality work, namely the University of Göttingen.

It is also thanks to his influence that she found a position as a lecturer in Stockholm, beginning in 1883. The two mathematicians maintained an active correspondence until KOWALEWSKAJA's death in 1891.

WEIERSTRASS published relatively little, since he was continually revising and expanding his ideas, often incorporating new results spontaneously into his lectures. He agreed to the publication of his lecture notes in his collected works, of which he was able to supervise the publication of the first two volumes (in what would eventually be a total of seven). But then, he suffered a rapid deterioration in his health. He spent the last three years of his life, when he was well enough, sitting in a wheelchair. He died in 1897, greatly honoured, of a lung infection.

Among the many mathematical results that are due to him, we might mention the following four representative examples:

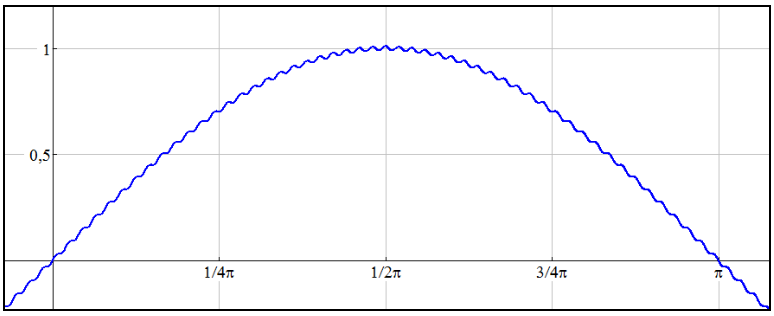
- In 1863, he proved that the field $\mathbb{C}$ of complex numbers is the only abelian extension of the set $\mathbb{R}$ of real numbers. (That is, the set of real numbers along with the usual operations of addition and multiplication with their associated laws can be extended to the set $\mathbb{C}$. In all other extensions of $\mathbb{R}$, the operation of multiplication is no longer commutative, for example in the field of quaternions, discovered in 1843 by WILLIAM ROWAN HAMILTON.)



- In 1860, he proved the result known today as the BOLZANO-WEIERSTRASS theorem: *Every bounded infinite sequence of real numbers has at least one accumulation point*.
- In 1872, he found a function that is everywhere continuous but nowhere differentiable, something that is counterintuitive.

Both facts had been discovered around 1817 and 1830, respectively, by BERNARD BOLZANO, something that became generally known to the world of mathematical research only in the twentieth century.

An example of such a WEIERSTRASSian monster curve is given by the function *f* defined by the following infinite series: $f(x) = \sin(x) + \frac{\sin(101x)}{100} + \frac{\sin(101^2 x)}{100^2} + \ldots$



- In 1885, WEIERSTRASS proved the following approximation theorem: *Every continuous function on a closed interval can be approximated arbitrarily closely by a polynomial.*

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